Head-On Collision between Two Pressure Jumps

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Conditions resulting from a head-on collision between two pressure jumps are considered. The atmosphere is assumed to consist of two incompressible layers and the motion to take place in one dimension only. In the final steady state it is found that two pressure jumps which are weaker and slower than the original jumps emerge from the region of collision. The process of collision results in a higher rate of loss of mechanical energy.

INTRODUCTION

It is a well-established fact that gravity waves of finite amplitude may occur in the atmosphere at a layer of inversion. These waves have been studied by a number of authors; see, for example, *Freeman* [1948, 1950] and *Abdullah* [1949]. A wave of elevation has been found to be doomed to break and propagate like a bore. *Tepper* [1950*a*] was the first to use the term pressure jump to designate a breaking wave and to associate it with the formation of squall lines. *Tepper* [1950*a, b*] also suggested that two pressure jumps may interact, either by overtaking or by intersection. He suggested that this interaction may, in some cases, be responsible for the genesis of tornadoes.

Various mechanisms may be deemed capable of forming two or more pressure jumps with different motions. It has been conjectured, for instance, that in the warm sector of a cyclone a pressure jump is created ahead of the cold front when the cold front acquires an acceleration. As mentioned by *Tepper* [1950*a*], there is no reason to believe that the cold front acquires the same acceleration at all its segments or even that the same segment may continue to move with the same acceleration. When the various segments acquire different motions, a pressure jump may radiate from each segment. These jumps may intersect with each other. When the same segment oscillates while advancing, jumps may be created corresponding to each forward motion, and successive jumps may overtake one another.

In the analytical discussion of the disturbances of the warm sector it has been the custom to assume the warm sector to be very wide and to study the effects of the motion of the cold front alone. Thus the warm sector is usually idealized as a long channel with one end open and the other closed by a moving plate. We know, however, that this does not represent the general case. The cold air ahead of the warm front closes this channel at the other end, and there is no reason to believe that the warm front remains inert. It is more likely that it may have a motion of its own which may create another family of disturbances that propagate backward into the warm sector. A more general idealized analogy is that of a channel of finite length, closed at both ends by two moving plates. We are therefore led to the study of the interactions between the waves produced by the two boundaries.

Another example of special interest may be found in closed systems like hurricanes. When a surge of cold air is allowed to penetrate into the circulation, it pushes the air on both sides as shown schematically in Figure 1. Two disturbances may be created simultaneously at the layer of inversion. These two disturbances move in opposite directions relative to the basic flow, and if they are not dissipated rapidly they are bound to meet and interact. Circulation that is restricted to tangential motion may be divided into separate cylindrical rings. If one of these rings is cut at the surge and opened up, it can again be reduced to the case of a straight channel of finite length whose ends are closed by two moving plates.

Figure 2 is a schematic representation of the idealized model in both of the examples. For the warm sector, the two fronts are idealized to two vertical plates. For the hurricane, the two edges of the surge are represented by the two vertical plates.

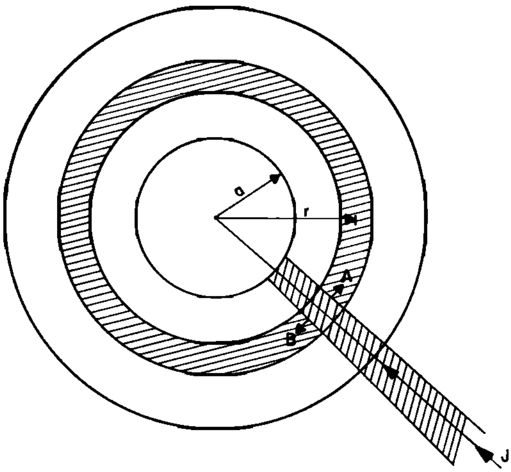

Fig. 1. Schematic representation of a surge of cold air entering a hurricane circulation. The inner circle represents the eye with radius a. r is the mean radius of some ring in the outer region affected by the surge. J is the re-entrant cold air A and B are the sides of the surge where it cuts the ring r.

It may be surmised that all possible kinds of waves are possible according to the prescribed motions of the two plates, and hence all possible interactions may result. In addition to its theoretical significance, a study of these interactions may prove to be of practical interest in the detailed explanation of some pertinent meteorological phenomena. In the present communication, however, discussion will be confined to the case of head-on collision between two pressure jumps. Such collision may be thought of as taking place in the warm sector when the cold front is accelerated while the warm front is retarded. In the closed circulation of a hurricane this would occur when a surge is introduced which pushes the air on both sides, or when a radial sector of the basic flow is allowed to vibrate relative to the preexisting flow. Either of these might cause conditions similar to those described in Figure 2.

The analogous phenomenon in gaseous flows is that of head-on collision between two shock waves. This has been studied extensively by a number of authorities (see, for example, *Courant and Friedrichs* [1948], *Howarth* [1953], or *Von Neumann* [1945]). For gases, however, it is usually assumed that energy is conserved. *Stoker* [1948] has shown that this law does not hold for bores; some energy is always lost. The same is true for the atmospheric pressure jump [*Abdullah*, 1954]. This reduces the number of equations at our disposal, but by making some reasonable but arbitrary assumptions we can reduce the number of unknowns so that a solution may be obtained.

The Idealized Model

Let the atmosphere be composed of two incompressible layers. The lower layer has a density ρ and moves in the positive direction with a constant velocity u_0 on the horizontal ground. Its initial thickness is h_0. The upper layer has a density ρ' and rests on the lower layer. Its free surface is at a height H, which is assumed to be much larger than h_0, so that it may remain undisturbed at all times. Let there be two very long vertical plates A and B (Figure 3) which are perpendicular to the direction of motion OX. The lower ends of these plates just touch the ground, and their heights are not much greater than h_0. The two plates confine between them a limited length of the lower fluid. Originally they may be moving with the same constant velocity of the lower layer, u_0.

If, in addition, the subsequent disturbed motion is assumed independent of the transverse direction, and taking place in the direction perpendicular to the two plates, the problem is reduced to that of a narrow channel bounded by two frictionless vertical walls and containing

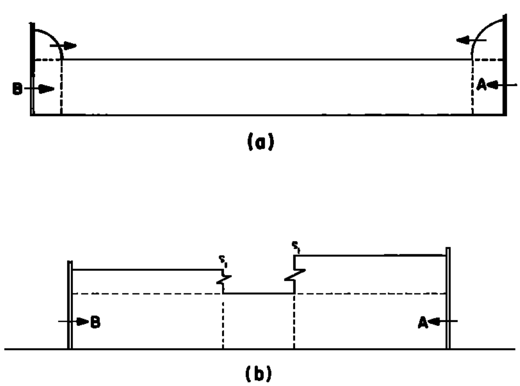

Fig. 2. Schematic representation of the idealized mechanism. AB is a straight channel closed by the two vertical plates A and B which are moving inward. Two humps are formed, ahead of the plates, which will break and form the two pressure jumps S_1 and S_2.

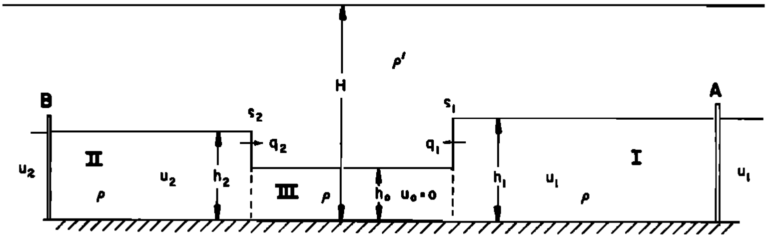

Fig. 3. The idealized model of two jumps S_1 and S_2 in a stratified two-layer incompressible atmosphere before collision.

two layers of fluid, the two plates being immersed in the lower fluid as shown in Figure 3.

At a certain instant let A and B be moved instantaneously to some final constant velocities u_1 and u_2, respectively. If u_1 is in the negative and u_2 in the positive direction, the subsequent conditions are expected to be as in Figure 3. The two pressure jumps S_1 and S_2 which are formed approach each other with the velocities q_1 and q_2, respectively. These velocities will be assumed to remain constant. The fluid particle velocities behind the two jumps are u_1 and u_2, respectively, and the heights of the upper surfaces of the lower layers are h_1 and h_2.

It is quite obvious that when the lower fluid is disturbed the upper fluid may be disturbed also. However, the effect of these disturbances on the motion of the lower fluid is neglected. This may be approximately realized if the pressure jumps are weak and if the height H is much larger than the height h.

A description of conditions after collision can be given qualitatively. The lower layer may be divided into three regions (see Figure 4). Region I contains the fluid between plate A and the fluid disturbed by the interaction. Here the fluid has its original height h_1 and velocity u_1. Region II contains the fluid between plate B and the middle region. It has the height h_2 and the velocity u_2. Region III, which lies in the middle, results from the mixing of the converging fluids under the colliding jumps. It is to be expected that a vorticity sheet will be created where the two fluids meet and where a velocity discontinuity may be observed. The fluids on the two sides may acquire different transverse and vertical motions. This state of affairs cannot last long, however. Because of turbulence and other frictional forces, and because of the lack of difference in densities, the two fluids are expected to mix. If transverse motion is forbidden, according to our original assumption, the mixed fluid will pile up at the middle region. A final steady state is then approached in which a uniform homogeneous mass results. It will be assumed that the upper surface of this new mass is horizontal and at height h_3 and that the velocity of its particles is u_3, which is assumed to be the same everywhere. Two new pressure jumps are formed at the two ends of the middle region, S_{13} and S_{23}. They advance backward into regions I and II. The problem is to obtain a description of these jumps.

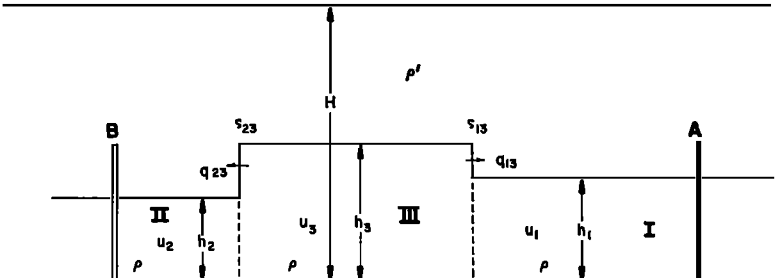

Fig. 4. The idealized model in the final steady state after collision. S_{13} and S_{23} are the two emerging jumps receding from each other with velocities q_{13} and q_{23}, respectively.

Description of a Single Pressure Jump

Stoker [1948] has derived the equations describing conditions at a single bore. The following is a slight extension of his method, which is meant to account for the existence of the upper layer in the case of a pressure jump.

Figure 5 is a schematic representation of the situation in an idealized model. The jump S is moving to the left with a constant velocity q. Disturbances in the upper layer are neglected, so that the height H of its free surface remains constant. The height of the interface behind the jump is h_1 and ahead of the jump it is h_0; the particle velocities are u_1 and u_0, respectively.

The conditions at the jump are two: conservation of mass and conservation of the rate of change of momentum. If there is no exchange of mass between the two layers, the conservation of mass holds for each layer separately. Since interest is centered around motion in the lower layer, this law will be derived for this layer only.

At the time t the jump may be at the point $\chi(t)$. Consider a column of the lower fluid bounded by two vertical planes consisting of the same fluid particles, $a_0(t)$ and $a_1(t)$, within which $\chi(t)$ lies. The law of conservation of mass for this column is

$$\frac{d}{dt}\int_{a_0(t)}^{a_1(t)} \rho h \, dx = 0 \qquad (1)$$

The rate of change of momentum is described as

$$\frac{d}{dt}\int_{a_0(t)}^{a_1(t)} \rho h u \, dx = \int_0^{h_0} p_0 \, dz - \int_0^{h_1} p_1 \, dz + \int_{h_0}^{h_1} p' \, dz \qquad (2)$$

where p_0 and p_1 are the pressures at points ahead of and behind the jump, respectively. The last term in p' represents the pressure force exerted by the upper layer on the protruding part of the jump.

If the basic assumption of long waves is carried here, so that the hydrostatic relation is assumed to hold, the following equations are found:

$$\begin{aligned} p_0 &= g\rho(h_0 - z) + g\rho'(H - h_0) & 0 \le z \le h_0 \\ p_1 &= g\rho(h_1 - z) + g\rho'(H - h_1) & 0 \le z \le h_1 \\ p' &= g\rho'(H - z) & h_0 \le z \le H \end{aligned} \qquad (3)$$

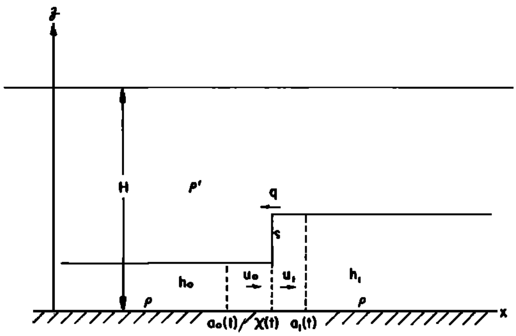

Fig. 5. Representation of a single pressure jump advancing under an inversion.

Upon following the usual method of integration (see *Courant and Friedrichs* [1948, p. 123] or *Stoker* [1948]) we get, from (1), the following condition when $a_0(t)$ and $a_1(t)$ are allowed to approach $\chi(t)$:

$$\rho h_1(u_1 - q) = \rho h_0(u_0 - q) \qquad (4)$$

Equation 2, in combination with equations 3, gives

$$\rho h_1 u_1(u_1 - q) - \rho h_0 u_0(u_0 - q) = \tfrac{1}{2}\rho g' h_0^2 - \tfrac{1}{2}\rho g' h_1^2 \qquad (5)$$

where

$$g' = g(\rho - \rho')/\rho \qquad (6)$$

Equations 4 and 5 are the required equations for the conditions to be satisfied by the lower fluid at the jump. They are identical with the corresponding relations for bores derived by *Stoker* [1948].

Combining (4) and (5) gives

$$(u_1 - q)(u_0 - q) = \frac{c_1^2}{2}\left(1 + \frac{u_1 - q}{u_0 - q}\right) \qquad (7a)$$

$$(u_1 - q)(u_0 - q) = \frac{c_0^2}{2}\left(1 + \frac{u_0 - q}{u_1 - q}\right) \qquad (7b)$$

where

$$c^2 = g'h \qquad (7c)$$

These equations will now be used for the discussion of the problem under consideration.

Descriptions of Jumps before Collision

Figure 3 is a schematic representation of the idealized situation before collision. Because the jump conditions remain invariant under translation with constant velocity, in accordance with the Galilean principle of relativity, no loss of generality is made by assuming the velocity of the undisturbed fluid to be zero. All velocities are measured relative to a frame of reference moving with the undisturbed fluid.

In the idealized model (Figure 3) there are two jumps, S_1 and S_2, moving toward each other with the velocities q_1 and q_2, respectively. The undisturbed fluid separating the two jumps is at a height h_0 and is at rest, so that $u_0 = 0$. The fluids behind the jumps are at heights h_1 and h_2, and the particles have the velocities u_1 and u_2, respectively.

Because it is normally easy to measure the velocity of a pressure jump, it will be assumed that q_1 and q_2 are known a priori. It is also assumed that the conditions in the undisturbed region are completely known. Once q_1, q_2, h_0, and u_0 are known, the other quantities, namely h_1, u_1, h_2, and u_2, can be determined from the jump relations.

Thus, if $u_0 = 0$ and if the quantity v is defined by

$$v = u - q \tag{8}$$

equation 7*b* takes the form

$$-qv_1 = c_0^2(1 - q/v_1)/2 \tag{9}$$

The conditions at the jump S_1 are

$$-q_1v_1 = c_0^2(1 - q_1/v_1)/2 \tag{10}$$

or

$$v_1^2 + c_0^2v_1/2q_1 - c_0^2/2 = 0 \tag{11}$$

Hence

$$v_1 = \frac{c_0^2}{4q_1}\left[-1 \pm \left(1 + \frac{8}{c_0^2} q_1^2\right)^{1/2}\right] \tag{12}$$

Equation 4 takes the form

$$h_1v_1 = -h_0q_1 \tag{13}$$

Hence

$$h_1 = -h_0(q_1/v_1) \tag{14}$$

Because q_1 has been assumed to be negative, and $h_0 > 0$, v_1 must be positive in order that h_1 remain positive. Again, because the magnitude of the radical in (12) is always greater than 1, the negative sign of the ambiguity must be chosen. This makes the quantity inside the brackets negative, and hence v_1 positive. Equation 12 may therefore be written as

$$v_1 = -\frac{c_0^2}{4q_1}\left[1 + \left(1 + \frac{8}{c_0^2} q_1^2\right)^{1/2}\right] \tag{15}$$

Equations 14 and 15 are the required equations for the jump S_1. The conditions at S_2 may similarly be found. They are

$$v_2 = -\frac{c_0^2}{4q_1}\left[1 + \left(1 + \frac{8}{c_0^2} q_2^2\right)^{1/2}\right] \tag{16}$$

$$h_2 = -h_0(q_2/v_2) \tag{17}$$

The last four equations serve to determine the four unknowns h_1, v_1, h_2, and v_2.

Conditions after Collision

Figure 4 is a schematic representation of the conditions after collision. When a steady state has been attained, the fluid may be considered in each of three separate regions. Region III forms at the middle, where the original jumps have interacted. The fluid comes to a steady state at a height h_3 and particle velocity u_3, both assumed constants. This region is bounded by two new jumps S_{13} and S_{23} which move with the constant velocities q_{13} and q_{23}, respectively. On the right, region I has the height h_1 and the particle velocity u_1. On the left, region II has the height h_2 and the particle velocity u_2. These latter quantities are the same as before collision. It is required to determine q_{13}, q_{23}, h_3, and u_3, quantities which describe the two emerging jumps completely.

In order to do so, we again use (4) and (7*a*), with the pertinent quantities. Thus the following equations describe conditions at S_{13}:

$$h_1(u_1 - q_{13}) = h_3(u_3 - q_{13}) \tag{18}$$

$$(u_1 - q_{13})(u_3 - q_{13}) = \frac{c_1^2}{2}\left(1 + \frac{u_1 - q_{13}}{u_3 - q_{13}}\right) \tag{19}$$

Similarly, the following equations describe conditions at S_{23}:

$$h_2(u_2 - q_{23}) = h_3(u_3 - q_{23}) \tag{20}$$

$$(u_2 - q_{23})(u_3 - q_{23}) = \frac{c_2^2}{2}\left(1 + \frac{u_2 - q_{23}}{u_3 - q_{23}}\right) \tag{21}$$

These are the four equations which must be solved simultaneously for the four unknowns.

From (18) the following is obtained:

$$u_3 - q_{13} = (u_1 - q_{13})h_1/h_3 \qquad (22)$$

Insertion in (19) yields

$$\frac{h_1}{h_3}(u_1 - q_{13})^2 = \frac{c_1^2}{2}\left(1 + \frac{h_3}{h_1}\right) \qquad (23)$$

from which the following is obtained:

$$u_1 - q_1 = -\frac{c_1}{\sqrt{2}}\left[\left(\frac{h_3}{h_1}\right) + \left(\frac{h_3}{h_1}\right)^2\right]^{1/2} \qquad (24)$$

The negative sign of the ambiguity is admitted because the jump is, presumably, forward facing. See *Stoker* [1948].

Similarly, the following equations for S_{23} may be obtained from (20) and (21):

$$u_3 - q_2 = (u_2 - q_{23})h_2/h_3 \qquad (25)$$

$$u_2 - q_2 = \frac{c_2}{\sqrt{2}}\left[\left(\frac{h_3}{h_2}\right) + \left(\frac{h_3}{h_2}\right)^2\right]^{1/2} \qquad (26)$$

where the positive sign of the ambiguity is admitted.

Upon elimination of u_3 between (22) and (25) the following equation is obtained:

$$(h_3 - h_1)q_{13} + h_1u_1 = (h_3 - h_2)q_{23} + h_2u_2 \qquad (27)$$

Substitution for q_1 and q_2 from (24) and (26) results in the following equation for h_3:

$$(h_3 - h_1)\frac{c_1}{\sqrt{2}}\left[\left(\frac{h_3}{h_1}\right) + \left(\frac{h_3}{h_1}\right)^2\right]^{1/2} + h_3u_1$$

$$= h_3u_2 - (h_3 - h_2)\frac{c_2}{\sqrt{2}}\left[\left(\frac{h_3}{h_2}\right) + \left(\frac{h_3}{h_2}\right)^2\right]^{1/2} \qquad (28)$$

Having found h_3 from (28), we can find q_{13} and q_{23} from (24) and (26), respectively. Knowing q_{13} and h_3, we can find u_3 from (22). Equation 25 serves as a check.

Numerical Example

Before computing the results for a numerical example, and in order to put the results in a more useful general form, we introduce the following dimensionless quantities:

$$u = u'c_0, \qquad q = q'c_0, \qquad v = v'c_0 \qquad (29)$$
$$c_i = c_i'c_0, \qquad h = h'h_0$$

where the primed quantities are dimensionless and the subscript 0 refers to the undisturbed region.

Insertion of these quantities in the equations describing conditions at the jumps yields the following set of equations in dimensionless quantities, the primes having been dropped because no confusion is expected.

Conditions before collision at S_1:

$$v_1 = -\frac{1}{4q_1}[1 + (1 + 8q_1^2)^{1/2}]$$
$$h_1 = -q_1/v_1 \qquad (30)$$
$$v_1 = u_1 - q_1$$

Conditions before collision at S_2:

$$v_2 = -\frac{1}{4q_2}[1 + (1 + 8q_2^2)^{1/2}]$$
$$h_2 = -q_2/v_2 \qquad (31)$$
$$v_2 = u_2 - q_2$$

Conditions after collision are described by (22), (24), (25), (26), and (28), which retain the same forms after the introduction of the dimensionless quantities.

Now consider the following representative numerical values for the various variables.

$T_0 = 294°$K, $T' = 300°$K, where T is the absolute temperature. Hence it follows from (6) that $g' = 20$ cgs units. Let $h_0 = 1.62 \times 10^5$ cm. Hence it follows from (9) that $c_0 = 1.8 \times 10^3$ cm sec^{-1}. Let $q_1 = -2.16 \times 10^3$ cm sec^{-1} and $q_2 = +1.98 \times 10^3$ cm sec^{-1}.

The corresponding dimensionless quantities are, from (29), $c_0' = 1$, $h_0' = 1$, $q_1' = 1.2$, and $q_2' = +1.1$.

From (30) and (31) the following values for the conditions before collision are obtained:

$v_1' = 0.945$, hence $u_1' = -0.255$

$h_1' = 1.27$

$v_2' = -0.970$, hence $u_2' = +0.13$

$h_2' = 1.135$

Upon inserting these values in (28) and setting each side of this equation to be equal to a variable y, we can solve this equation graphically for h_3' as shown in Figure 6. The value of h_3' as read from this figure is 1.42.

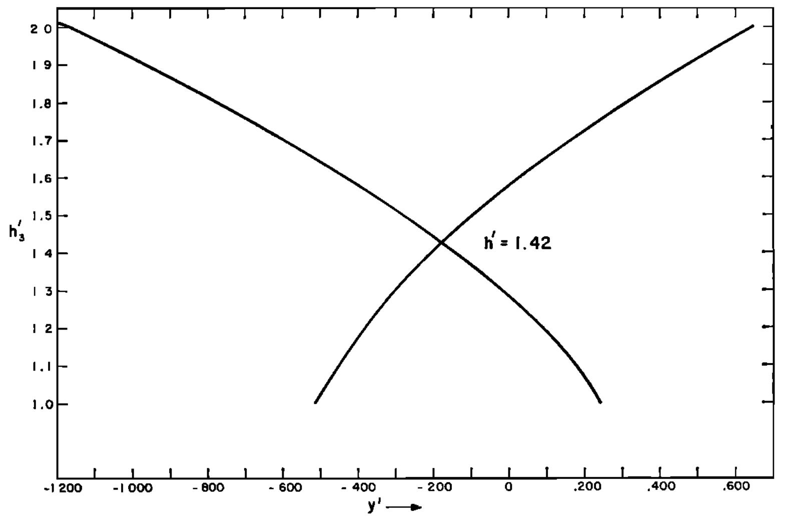

Fig. 6. Graphical solution of equation 28. The ordinate is the unknown height h_3' in dimensionless units and the abscissa is a quantity y'.

Upon using this value in (24) and (26), and then in (22) and (25), we get the following values for the elements after collision: $q_{13}' = +0.965$, $q_{23}' = -1.129$, and $u_3' = -0.122$. These quantities describe conditions after collision completely.

The corresponding physical quantities are the following.

Before collision:

$$h_1 = 2.06 \text{ km}, \ u_1 = -4.6 \text{ m sec}^{-1} \qquad \text{at } S_1$$

$$h_2 = 1.84 \text{ km}, \ u_2 = 2.34 \text{ m sec}^{-1} \qquad \text{at } S_2$$

After collision:

$$h_3 = 2.3 \text{ km}, \quad u_3 = -2.2 \text{ m sec}^{-1}$$

Velocity of S_{13}:

$$q_{13} = 17.38 \text{ m sec}^{-1}$$

Velocity of S_{23}:

$$q_{23} = -20.3 \text{ m sec}^{-1}$$

From the last two quantities we see that the emerging jumps are slower than the original jumps.

The strength of a pressure jump is usually measured by the ratio of the excess of height to the smaller height. Thus

$$\sigma = \Delta h/h_i \tag{32}$$

where σ is the strength of the jump, Δh is the difference in height between the levels of fluid on the two sides, and h_i is the height of the lower fluid. Upon using this criterion we see that the strengths of the jumps are the following.

Before collision:

$$\sigma(S_1) = 0.270/1 = 0.270$$

$$\sigma(S_2) = 0.135/1 = 0.135$$

After collision:

$$\sigma(S_{13}) = 0.15/1.27 = 0.118$$

$$\sigma(S_{23}) = 0.285/1.135 = 0.250$$

Thus it is seen from these values that the result of collision between two pressure jumps is two weaker jumps. This result agrees with the analogous result of collision between two shocks in a gas. See *Howarth* [1954, p. 137].

Figure 7*a* shows the conditions before collision pertinent to the numerical values considered in the present section. Figure 7*b* shows conditions after collision. Figure 8 is a plot of the paths of the jumps. State 0 corresponds to the nondisturbed flow. State I corresponds to region I, state II to region II, and state III to the fluid at the middle separating the two jumps after collision. The paths of the particles are also shown in the three regions.

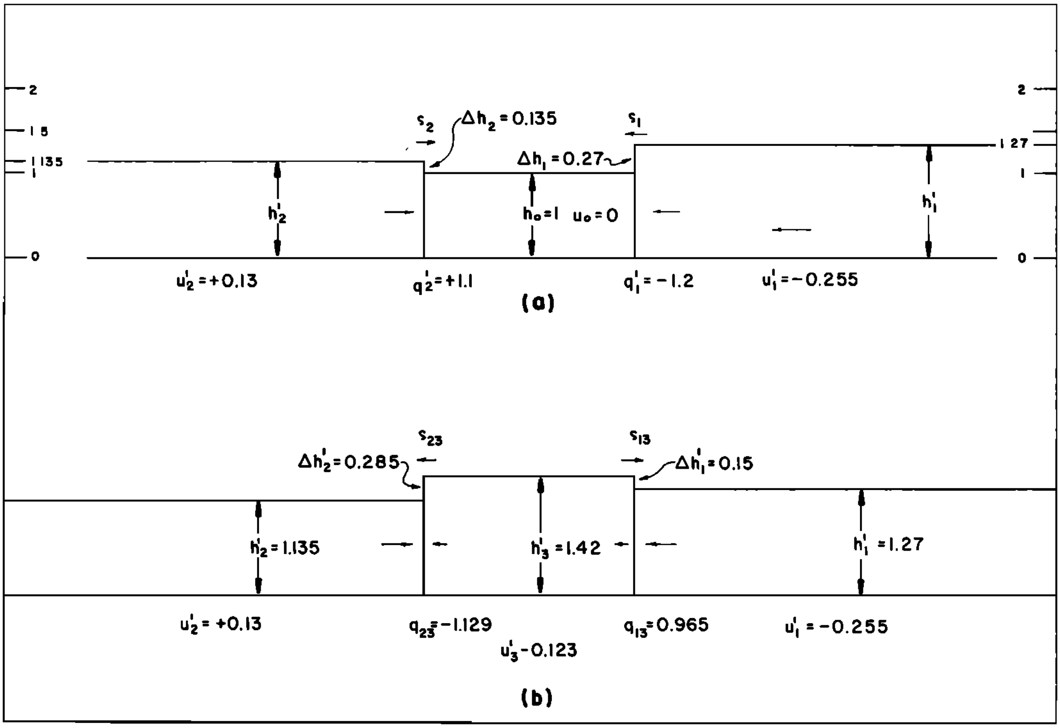

Fig. 7. (*a*) A plot of the numerical example before collision. All quantities are in dimensionless units. (*b*) Conditions after collision.

ENERGY

It has been shown by *Abdullah* [1954] that Stoker's formula for the rate of loss of energy at a bore is also valid for a pressure jump. Thus the rate of loss of energy at a unit length of the jump line is

$$\frac{dE}{dt} = \frac{mg'}{\rho} \frac{(\bar{p}_0 - \bar{p}_1)^3}{4\bar{p}_1\bar{p}_0} \tag{33}$$

where $m = \rho hv$ and $\bar{p} = \rho h$.

Upon introducing the dimensionless quantities defined in (29), we see that (33) takes the form

$$K\, dE/dt = v'(\Delta h')^3/h' \tag{34}$$

where $K = 4/c_0^2\, \rho h_0 =$ a constant.

Equation 34 may now be used for a comparison of the rates of losing energy at the jumps, before and after collision.

Substitution of the relevant values into (35) gives the following rates of loss of energy.

Before collision:

$$K \left.\frac{dE}{dt}\right)_{S_1} = 0.01850 \qquad \text{at } S_1$$

$$K \left.\frac{dE}{dt}\right)_{S_2} = 0.00141 \qquad \text{at } S_2$$

Total rate before collision

$$= 0.01991$$

After collision:

$$K \left.\frac{dE}{dt}\right)_{S_{13}} = 0.0029 \qquad \text{at } S_{13}$$

$$K \left.\frac{dE}{dt}\right)_{S_{23}} = 0.0205 \qquad \text{at } S_{23}$$

Total rate after collision

$$= 0.0239$$

It is therefore seen that the two jumps emerging after collision lose energy at a greater rate than the original jumps before collision.

If the pressure jumps are supposed to feed on the mechanical energy of the storm in which they are imbedded, it follows that the emerging jumps consume energy faster than the original. Hence the storm is more quickly dissipated

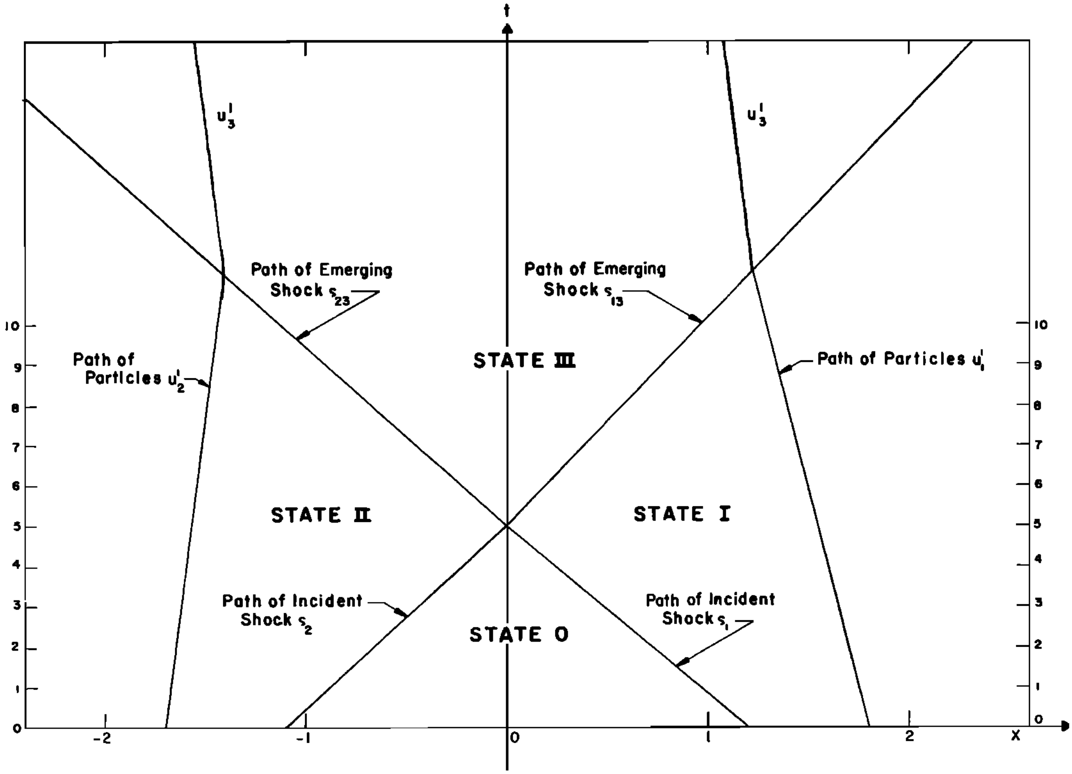

Fig. 8. A schematic plot of the paths of the two shocks before collision, S_1 and S_2, and after collision, S_{13} and S_{23}. The paths of the particles in the three distinct regions are indicated.

when collision takes place between its pressure jumps.

Conclusions and Further Remarks

In the light of the discussion above, it may be inferred that atmospheric pressure jumps may interact in an analogous way to the interaction between shock waves in gases. The result of a head-on collision between two pressure jumps is the emergence of two jumps, each of which is weaker and slower than its parent jump. The rate of loss of energy increases after collision, a result which shows that the process helps to dissipate the energy of the storm in which the jumps are imbedded. This, of course, follows only under the assumption that no energy sources are involved.

In conclusion, the following remarks may be made: (1) In the mathematical analysis the effect of disturbances in the upper layer has been neglected. This may be approximated only if the pressure jumps are weak and the upper layer is so deep that its free surface may remain nearly unaffected. A better approximation can be obtained by studying the interaction between disturbances in the two layers. (2) The transient state resulting immediately upon collision has not been discussed in the present article. The final steady conditions after a homogeneous fluid has been formed in the middle region are treated. There is room here for further development of the theory.

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