THE MERIDIONAL GROWTH OF SQUALL LINES

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(Manuscript received 30 November 1953)

ABSTRACT

Squall lines that form ahead of an accelerating cold front are sometimes observed to grow in length. In this paper an attempt is made to explain this interesting property of squall lines, and to deduce a mathematical formula by which this growth may be forecasted.

The idealized model is based on the following assumptions: (1) the ground on which the flow occurs is horizontal and devoid of any elevations; (2) there is an inversion layer above the lower surface flow; (3) the acceleration of the cold front is constant; (4) no lateral velocities are produced because of the motion of the cold front.

On the basis of these assumptions, and with use of the method of characteristics, a formula is derived for the formation and growth of a squall line. A numerical example is given to clarify the method.

1. Introduction

It is a matter of common observation that some middle-latitude cyclones develop squall lines in their warm sectors. These squall lines appear early in the life of the occluding cyclone, and propagate ahead of the cold front. Because of the severe weather that usually accompanies a squall line, the interest of many investigators has been directed towards its detailed study. Tepper (1950) has advanced the proposition that a squall line is caused by a propagating pressure jump. He suggested that the advancing cold air, behind the cold front, acts as a piston pushing the warm air ahead of it. If there is a layer of inversion in the warm sector, the warm air rises in the form of a bump in the immediate neighborhood of the cold front. The "wave" thus produced travels downstream as a gravitational "wave" which will eventually break, creating a jump in the flow. Because of the breaking of the fluid, a considerable amount of mechanical energy is released (Abdullah, 1949). This energy manifests itself in the form of turbulence and may be enough to overcome the stability of the air caused by the inversion. If the air of the warm sector is originally conditionally unstable, the released mechanical energy of the jump may be enough to release the latent instability of the overlying air. This gives rise to the convective activity which is associated with a squall line. The pressure jump, therefore, acts as a trigger to initiate the squall lines of severe weather. Moreover, because of the regularity inherent in the motion of a jump, the jump gives the associated squall line the order that is usually observed in its motion.

When the advancing cold front begins to decelerate, a "depression" wave advances behind the jump, along the surface of the inversion layer. This depression wave finally catches up with the jump and ultimately destroys it.

More recent work carried out by Tepper (1952) bears out these ideas. In that work, a network of micro-observations was used. The barographs and the anemographs clearly bring out the discontinuous character of the weather elements associated with a squall line.

One very interesting investigation in this connection was carried out by Brunk (1949). Brunk was mainly interested in describing the behavior of the "pressure pulsations" of 11 April 1944, which propagated eastward across the continent. He noticed that the synoptic conditions occurring with the pulsation created a squall line in the warm sector of the cyclone in which the pulsation formed at its northeastern sector.

Careful study of the isochrones of Brunk's squall line reveals a very interesting feature of these atmospheric discontinuities. Thus, it can be seen from fig. 8 of Brunk's paper that the squall line grew in the meridional direction as it proceeded eastward. It can be seen from that figure that the length of the squall line was about 100 mi at 2130 CST on 10 April, when it was first recorded. Its length grew steadily thereafter, until it became about 500 mi at 1330 CST 11 April. During this period, the squall line advanced eastward a distance of about 700 mi. Fig. 12 of the same paper shows the continuous growth of the lines of the associated thunderstorm activity.

It thus appears that the squall line that precedes a cold front sometimes grows in the meridional direction.

[1] The research reported in this paper has been sponsored by the Geophysics Research Directorate, Air Force Cambridge Research Center, under Contract AF19(122)-49.

When it does so, it seems to start in a very limited region, usually at the left side of the basic flow in the warm sector. Then it grows by extending in the meridional direction, usually toward the right side of the flow.

It is the purpose of the present paper to bring out this interesting property of squall lines, and to show that this growth is a necessary characteristic of these phenomena. The rate and extent of growth may vary from one individual case to another. An attempt is made to present a theoretical explanation of this property, and to predict the growth of the isochrones. It is believed that such a study may prove to be of some use in forecasting the weather that comes with these severe storms.

2. Some qualitative considerations

The basic concepts of the mechanism of the process by which the jump is formed have been explained by various writers. See, for instance Freeman (1948) or Abdullah (1949). The main features of this process may be summarized as follows.

In the idealized model, it will be assumed that there exists a current of homogeneous air that flows in a straight path with a constant velocity over horizontal level ground. On top of this current, there is a mass of lighter air. The air of the two layers may be considered as incompressible. The surface that separates the two masses may be a sharp surface of inversion. This surface will here be referred to as the upper surface of the lower layer. The upper air mass will be referred to as the upper layer. The upper layer will be assumed to have a free surface, whose height may be taken as the height of the homogeneous atmosphere. All disturbances may be assumed to take place in the lower layer. The free surface of the upper layer may, therefore, remain undisturbed at all times.

The cold air, behind the cold front, will be assumed to be moving initially with the same speed as that of the current in the warm sector and in the same direction. The cold front will be assumed vertical, and it advances forward like a non-permeable wall. Because the picture portrayed here is similar to the problem of pushing a vertical plate into a stream of water, the cold front is sometimes referred to as the "plate." Its path is sometimes referred to as the "plate path."

Because of the rotation of the earth, the upper surface of the lower layer is not horizontal. In general it has a definite slope. In a homogeneous air mass, like the one postulated here, the slope is mainly determined by the fields of motion in the two air masses. If the motion is assumed to be strictly horizontal, and the speed of the upper flow, in the direction of the lower flow, is either less or equal to that of the lower flow, it can be shown that the surface slopes downwards towards the center of the cyclone. To an observer moving with the current, the lower layer is deeper to his right hand side than it is to his left hand side. As the cold front starts to be accelerated with respect to the fluid ahead of it, the fluid of the lower layer in its immediate neighborhood is raised in the form of a bump. This bump advances in the form of a "wave." The whole fluid underneath the wave is disturbed and attains an additional velocity forward. The velocity of progress of a point on the "wave form" is an increasing function of its height. The uppermost parts of the wave form, therefore, advance faster than the parts beneath them. This will result in the continuous steepening of the slope of the forward edge of the wave and its eventual breaking.

Because the upper surface of the lower layer is not horizontal but has a definite slope in the meridional direction, the wave created by the elevated fluid does not advance parallel to the cold front. The wave over the deeper fluid advances faster. Under the conditions mentioned in the previous paragraph, where the inversion layer slopes downward towards the center of the cyclone, the right-hand side of the wave advances faster. Moreover, because of the slope of the surface, the breaking does not take place at the same time at all points of the forward edge of the wave form. Under the above conditions, the left-hand side breaks first, as will be shown below. If the basic current in the warm sector is westerly, and the surface of inversion slopes downward towards the center of the cyclone, the breaking will start in the northern part of the current first. The breaking will thereafter proceed southward. If the squall line is associated with the jump resulting from the breaking, it immediately follows that the squall line will appear to grow in the windward direction. In this case, the growth proceeds southward.

3. Mathematical analysis

Let a Cartesian system of coordinates (X, Y, Z), which is fixed to the ground, be chosen in such a way that the X-axis coincides with the undisturbed position of the right edge of the lower current in the warm sector. Let it point in the positive direction of the undisturbed current. Let the Y-axis coincide with the position of the cold front just before it is accelerated, and let it point toward the center of the cyclone. The $X-Y$ plane is on the ground. Let the Z-axis be vertical and positive upward.

Let the velocity of the undisturbed current of the warm sector be u_0, the density of the air mass in this current be ρ, and let the density of the lighter air on top of it be ρ', both air masses being assumed incompressible.

The undisturbed flow in the warm sector is governed by the equations

$$0 = -P_x/\rho, \; fu_0 = -P_y/\rho, \text{ and } g = -P_z/\rho, \quad (1)$$

where f is the Coriolis parameter, assumed a constant in the present analysis, g is the acceleration of gravity, and the subscripts denote partial differentiations.

It follows from (1) that the pressure on the ground decreases linearly with increasing y, so that the following relation holds:

$$P = P_0 - \rho f u_0 Y, \quad (2)$$

where P_0 is the ground pressure at the X-axis.

If the upper layer is at rest, or if it flows with a constant velocity in the X-direction, it is obvious that the upper surface of the lower layer is a plane which has an equation of the form

$$mY + Z = h_1, \quad (3)$$

where m is the constant slope of the upper surface of the lower layer, and h_1 is the height of the lower layer above the X-axis. If a is the width of the lower current, and h_2 is the height of the upper surface above the line $Y = a$ (see figs. 1 and 2), m is given by

$$m = (h_1 - h_2)/a. \quad (4)$$

The slope m may also be computed from dynamical considerations, under the proper assumptions. This will be shown below.

To simplify matters, it will be assumed that the velocity in the Y-direction is always negligible. This assumption is admittedly crude and may only be justified by the simplification it introduces in the mathematical treatment of the problem under consideration. If the transverse accelerations are taken into account, the problem becomes a three-dimensional one for which no exact mathematical solution is known yet. (See Courant and Friedrichs, 1948.)

By virtue of this assumption, it follows that the elevated fluid is assumed not to overflow laterally in the Y-direction. Hence, a particle that has initially the ordinate Y_i will preserve this same ordinate in its subsequent motion. The fluid may, therefore, be subdivided into an infinite number of thin vertical strips. Each strip may now be considered independently, since its state is not affected by the neighboring strips.

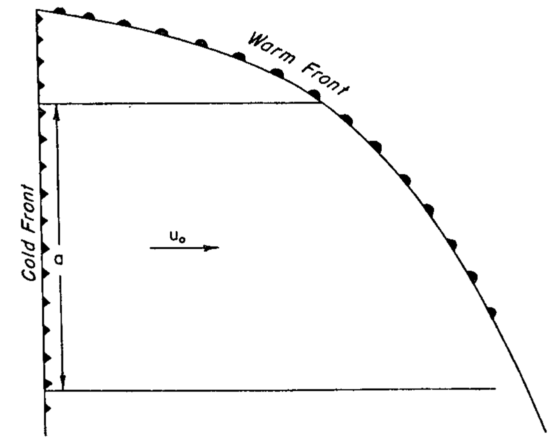

FIG. 2. Horizontal cross-section of undisturbed current of warm sector.

Let a frame of moving coordinates (x, y, z) be so chosen that it travels parallel to the X-axis with the constant velocity u_0. At the instant $t = 0$, the moving frame coincides with the stationary system of coordinates. The relation between the coordinates of a point relative to the two frames may be given by

$$X = x + u_0 t, \quad Y = y, \quad \text{and} \quad Z = z, \quad (5)$$

To an observer moving with this new frame, the undisturbed fluid is at rest. The slope of the upper surface of the lower layer is not affected; but the velocities in the X-direction are reduced by the amount u_0. There is no other effect, as long as the earth is considered flat and its rotation is neglected.

Before we write down the equations governing the motion of the disturbed fluid, a word may be added about the role of the Coriolis force in the problem under consideration. According to the assumptions made in the present paper, the Coriolis force acts on the undisturbed fluid only. It, therefore, serves to establish the basic current with the basic undisturbed slope of the upper surface of the lower layer. Any effect that may result later will be neglected. For the

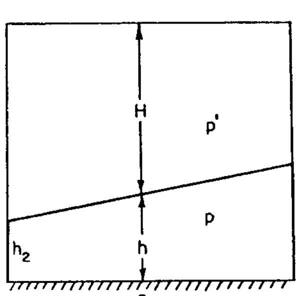

FIG. 1. Vertical cross-section of undisturbed current of warm sector.

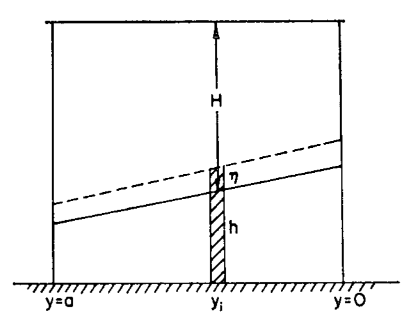

FIG. 3. Schematic representation of disturbed fluid. Shaded area represents ith strip, and dotted line represents elevated surface of lower layer.

disturbed motion, the earth may, therefore, be considered as stationary and flat.

Now consider any strip situated at the ordinate y_i. Let the undisturbed height of the upper surface of the lower layer at this strip be h. Let the height of the free surface of the upper fluid above the strip be H. When the fluid is disturbed, let the elevation of the fluid above the upper surface of the lower layer be η (see fig. 3). The free surface of the upper fluid may be assumed to remain undisturbed at all times.

The equation of continuity relative to the moving system of coordinates is

$$u_x + w_z = 0, \tag{6}$$

where w is the vertical component of the velocity, and u is the horizontal velocity in the x-direction.

The boundary conditions are the following: (1) the vertical velocity must vanish at the ground; thus, $w = 0$ at $z = 0$; and (2) the fluid particles that are initially at the upper surface of the lower layer shall remain there throughout the motion; thus, $dF/dt = 0$, where $F(x,y,z,t) = 0$ is the equation of the disturbed surface.

Upon integrating (6) with respect to z from the ground to the disturbed upper surface of the strip, and introducing the two boundary conditions, we obtain (see Stoker, 1948)

$$\eta_t = -\left[\int_0^{h+\eta} u\,dz\right]_x. \tag{7}$$

The equation of motion is

$$u_t + u\,u_x = -\,p_x/\rho. \tag{8}$$

If the vertical accelerations are neglected, the pressure may be given by the hydrostatic equation, as follows:

$$p = g\rho'[H(y) - \eta(x,y,t)] + g\rho[h(y) + \eta(x,y,t) - z]. \tag{9}$$

Equation (9) yields the following relation:

$$p_x = g(\rho - \rho')\eta_x.$$

Upon introduction of the quantity c, defined by

$$c^2 = g'(\eta + h) \quad \text{and} \quad g' = g(\rho - \rho')/\rho, \tag{10}$$

the equations of continuity and motion take the following forms:

$$\begin{aligned} 2c_t + 2u\,c_x + c\,u_x &= 0, \\ u_t + 2c\,c_x + u\,u_x &= 0. \end{aligned} \tag{11}$$

The quantity c is the velocity of progress of a point on the wave form relative to the disturbed fluid in the moving frame of axes (see Stoker, 1948). Because both η and h are functions of y, it follows that c is a function of y also. Since, under the previous assumptions, h decreases with increasing y, it follows that the effect of the variation of h is to make c a decreasing function of y. Thus, if a straight wave front is considered, this effect will make the wave front rotate in such a way that its right-hand side recedes from the cold front faster than its left-hand side. If the basic current of the lower layer is westerly, this rotation is counterclockwise. If the slope m is negative, such that the lower layer is deeper toward the center of the cyclone, the wave front rotates in a clockwise direction.

Because of the exact similarity between (11) and the corresponding equations of gravitational waves of finite amplitude, the results of the latter problem may now be adopted without going into their actual derivations. For these derivations, the reader may be referred to previous works (Stoker, 1948).

From the results of the problem of gravitational waves, it follows that the families of characteristics for each strip are given by the following equations:

1. On the family C_1, the slope is given by $dx/dt = u + c$, and the following relation holds on these characteristics:

$$u + 2c = k_1, \text{ a constant}; \tag{12}$$

2. On the family C_2, the above relations take the following forms: $dx/dt = u - c$, and

$$u - 2c = k_2, \text{ a constant}. \tag{13}$$

Because c decreases with y when m is positive, the slopes of the characteristics C_1 on the x-t plane in the x-y-t space decrease with increasing y.

Since, relative to the moving observer, the wave is progressing into still fluid, it follows that the wave is a case of simple waves. Therefore, the characteristics C_1 are a family of straight lines. If the time is measured from the instant when the cold front starts accelerating, the straight characteristic C_1^0 of the leading point in the wave form, whose elevation is $\eta = 0$, is given by

$$dx/dt = c_0, \tag{14}$$

where $c_0^2 = g'h$.

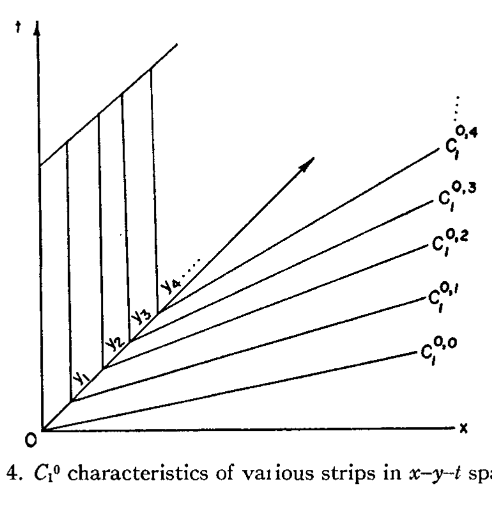

FIG. 4. C_1^0 characteristics of various strips in x–y–t space.

In the x-y-t space, the totality of the C_1^0 characteristics for all the vertical strips form a skewed ruled surface, as shown in fig. 4, in which m was taken to be positive. In this figure, the first upper superscript denotes time and the second denotes the ordinate y_i.

It is seen from this figure that the slope of the characteristics $C_1^{0,i}$ decreases with increasing y.

The slope of any straight characteristic issuing from the t-axis at a later instant τ may be given by

$$\frac{dx}{dt} = \tfrac{3}{2}u(\tau) + c_0. \tag{15}$$

Consider the x-t plane for the ith strip. In this plane, the path of the cold front may be represented by the plate curve (see fig. 5). If the front is moving forward with respect to the moving frame of axes, with the constant acceleration α, the equation of this curve is

$$x = \tfrac{1}{2}\alpha t^2. \tag{16}$$

Fig. 5 shows the straight characteristics C_1^i issuing from the plate curve. These characteristics intersect. They also have an envelope, whose cusp lies on the $C_1^{0,i}$ line (see Courant and Friedrichs, 1948). The cusp gives the location, in the x-t plane, of the point where the slope of the forward edge of the wave becomes infinite. At this point, the breaking starts in the ith strip.

It has been shown in a previous paper (Abdullah, 1949) that the time of breaking, for the case under consideration, is given by

$$t_{0i} = \tfrac{2}{3}\frac{c_{0i}}{\alpha}. \tag{17}$$

The position of the point where the breaking takes place is given by

$$x_{0i} = \tfrac{2}{3}\frac{c_{0i}^2}{\alpha}. \tag{18}$$

This last result follows directly from (14) and (17), the cusp of the envelope being located on the characteristic C_1^{0i}.

Upon substitution in (17) and (18) from $c_0^2 = g'h$, and use of $h_i = h_2 + my_i$ for h, the following equations are obtained for the time and position of breaking in the ith strip:

$$\begin{aligned} t_{0i} &= \frac{2}{3\alpha}\left[g'(h_2 + my_i)\right]^{\frac{1}{2}}, \\ x_{0i} &= \frac{2}{3\alpha}g'(h_2 + my_i). \end{aligned} \tag{19}$$

If α does not depend upon y (in other words, if the cold front moves parallel to itself) and if m is positive, it follows from (19) that the breaking starts first at the left side of the current at the instant t_{02}, given by

$$t_{02} = \frac{2}{3\alpha}(g'h_2)^{\frac{1}{2}}.$$

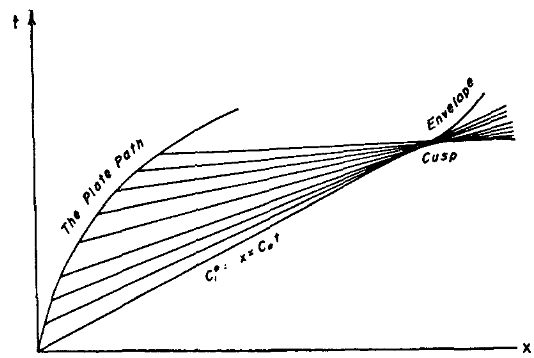

FIG. 5. Plate path and formation of cusp of envelope in x–t plane.

The breaking spreads towards the right-hand side of the lower current and reaches the point $y = 0$ at the instant t_{01}, given by

$$t_{01} = \frac{2}{3\alpha}(g'h_1)^{\frac{1}{2}}.$$

The positions of the points where the breaking first takes place may be found from the second of the relations (19).

The broken fluid travels as an atmospheric "jump" which initiates the squall line. However, it may very well be that the degree of stability of the atmosphere above the inversion layer is such that the energy released by the initial breaking is not enough to start the squall line. In this case the actual convective activity starts at a later instant, when the breaking has proceeded to an advanced degree.[2]

If the length of the squall line is denoted by l, it is seen that at the time t_{01} this length is zero. In other words, the squall line starts at a single point, the length l being the projection of the sloping length of the jump on the x-y plane. The subsequent length of the squall line may be found from the first of equations (19). Thus, at a later instant t, the length is given by

$$l = \frac{1}{m}\left(\frac{9\alpha^2}{4g'}t^2 - h_2\right). \tag{20}$$

[2] The writer is indebted to Dr. Tepper for this remark.

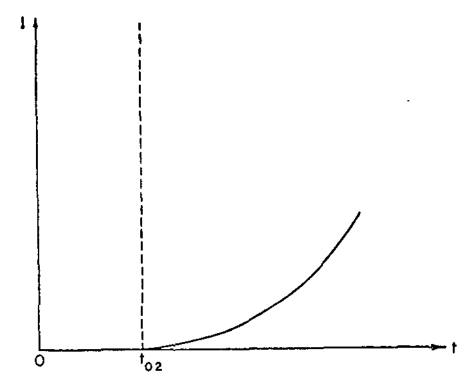

FIG. 6. Relation between length of squall line and time after plate starts pushing.

This equation is represented graphically by the continuous line in fig. 6. It has been assumed that the instability of the atmosphere is such that the squall line is started by the breaking when it first occurs. The rate of growth of the length of the squall line is found from (20), by differentiation with respect to time. Thus,

$$\frac{dl}{dt} = \frac{9\alpha^2}{2g'm}t. \tag{21}$$

The relation between the length of the squall line and its position relative to the moving frame of axes, in other words, relative to the moving warm sector, is given by the following equation, which follows from the second of equation (19):

$$x = \frac{2}{3\alpha}g'(h_2 + ml). \tag{22}$$

This relation is plotted in fig. 7. It is seen from this figure and from (22) that the length increases linearly with the distance traveled by the squall line relative to the warm sector.

The relation between the length of the squall line and the stationary frame of coordinates, which is fixed on the ground, is given by the following equation, which follows from (22) and (6):

$$X = \frac{2u_0}{3\alpha}[g'(h_2 + ml)]^{\frac{1}{2}} + \frac{2}{3\alpha}g'(h_2 + ml). \tag{23}$$

This equation is shown graphically in fig. 7 by the dotted curve.

Equations (20) to (23) give all the information required about the growth of a squall line which is formed under the conditions assumed in the present paper. These equations may also be used as a first approximation in forecasting the formation, location and meridional growth of a squall line.

On the basis of these equations and the preceding discussion, the following method of forecasting may be described:

1. Determine the height of the upper surface of the lower current from the aerological observations; from these determine the slope m; determine the acceleration of the cold front, α, from the movement of the cold front;

2. Spot the first point where the squall line starts, point A in fig. 7; this point may also be computed from the formulas given above, under the assumption that the first breaking is sufficient to release the convective instability; draw a straight line, AB, passing through the point A and parallel to the mean flow in the warm sector; this line defines the northern edge of the area that may be affected by the squall line during its subsequent motion; if the squall line is first spotted as a line of appreciable length, the straight line, AB, must be drawn through its left end, which is usually its northern edge;

3. Determine the position of the warm sector at the time for which the forecast is required; according to the assumptions made in the present paper, the warm sector may be considered as if it is propagating with the constant velocity of the basic current u_0; compute the slope of the line AC from (22), and draw this line to pass through the point A; this line determines the location of the southern end of the squall line; the location of the squall line at the instant for which the forecast is required may also be computed from (20) and (22); the region inside the triangle ABC is the region inside the warm sector which is probably affected by the squall-line disturbance. The curve A′C, computed from (23), determines the position on the ground of the southern edge.

This method may be clarified further by the following example.

4. Numerical example

Let the mean position of the warm sector be located at latitude 40°N. Let the pressure gradient in the warm sector be 1.0 mb/100 km, directed toward the north. This gradient corresponds to a westerly current of a mean velocity of 10 m/sec. Let the average width of the warm sector be 500 km. Let the height of the inversion layer above the center of the low be 1 km, and its height above the southern edge of the warm sector be 1.5 km. From the temperatures and pressure below the inversion and above it, the mean densities of the two layers may be computed. Let these be 1.1×10^{-3} and 1.05×10^{-3} g/cm³, respectively. Let the cold front start to be accelerated at the instant $t = 0$ with a constant acceleration of 0.05 cm/sec². It is required to determine the area that may be affected by the squall line.

In terms of the notation used, the data are the following:

$$\begin{aligned}
\partial p/\partial y &= 1.0 \text{ mb}/100 \text{ km};\\
u_0 &= 10 \text{ m/sec},\\
\rho &= 1.21 \times 10^{-3} \text{ g/cm}^3,\\
\rho' &= 1.16 \times 10^{-3} \text{ g/cm}^3,\\
f &= 0.937 \times 10^{-4} \text{ sec}^{-1},\\
h_1 &= 1.5 \text{ km},\\
h_2 &= 1.0 \text{ km},\\
\alpha &= 5 \times 10^{-2} \text{ cm/sec}^2,
\end{aligned}$$

and let

$$g = 980 \text{ cm/sec}^2.$$

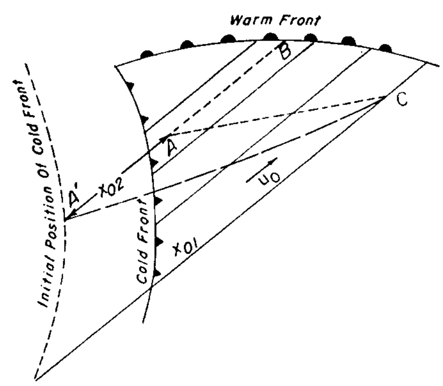

FIG. 7. Schematic illustration of method of forecasting. Line AB marks upper end of squall line, and line AC its lower end relative to moving warm sector. Arrow indicates basic current of warm sector. A′C is curve of growth relative to stationary frame of coordinates.

From these data and the proper equations,

$$m = 2 \times 10^{-3}, \qquad g' = 40.5 \text{ cm/sec}^2,$$
$$C_{02} = 20.1 \text{ m/sec},$$
$$C_{01} = 28.5 \text{ m/sec}.$$

From (17) it follows that the breaking starts in the northern part of the warm sector at the time t_{02}, which has the value

$$t_{02} = 26.8 \times 10^3 \text{ sec}, \; = 7^{\text{hr}}\, 27^{\text{min}} \text{ after the cold front started to accelerate.}$$

The breaking reaches the southern part of the current of the warm sector at the time t_{01}, which has the value

$$t_{01} = 38 \times 10^3 \text{ sec}, \; = 10^{\text{hr}}\, 33^{\text{min}} \text{ after the cold front started to accelerate.}$$

The length of the squall line was zero at t_{02}, and it became 500 km at t_{01}. The process of breaking took 3 hr and 6 min to spread from the northern end to the southern end.

The distance of the point A, where the breaking first started, from the initial position of the cold front relative to the warm sector is, from (19), $x_{02} = 539$ km.

The distance, inside the warm sector, between the position of the final breaking and that of the initial position of the cold front is $x_{01} = 777$ km.

During the first interval of time, t_{01}, the cold front has moved into the warm sector a distance of 180 km, so that the squall line started at the point A which is 359 km ahead of the cold front. By the time t_{02}, the cold front has moved into the warm sector a distance of 361 km, so that the squall line attained its full length at a distance of 416 km.

The area, on the fixed ground, influenced by the squall line is that bounded by the straight line A′B and the curve A′C. The latter curve has the following equation, which follows from (23): $X = 1.33 \times 10^4 \times [40.5(10^5 + 0.002)]^{\frac{1}{2}} + 54.0 \times 10(10^5 + 0.002)$ cm.

If the squall line, after attaining its maximum length, lies within the warm sector behind the warm front, it will move thereafter preserving this maximum length. The southern boundary of the affected area will be a broken line.

5. Decay of the squall line

It was stated in the introduction that the squall line starts to decay if it is overtaken by a depression wave. This depression wave may be caused by the slowing down of the cold front. Thus, the cold front which creates the squall line destroys it later.

The slowing down of the cold front may be in the form of a continuous decrease in the acceleration until it becomes zero, so that the cold front attains a certain maximum velocity and then is propagated with that same velocity. It may also be that the cold front may start to recede relative to the warm sector.

Because the problem of the decay of a "jump" is one which is somewhat different from that of the creation of a "jump," the writer refrains from treating it here. The reader may be referred to other works on this subject, such as that of Freeman (1948). However, it may be remarked here that the depression waves may be responsible for creating pressure pulsations of the nature of those observed by Brunk.

6. Discussion and further remarks

It may be inferred, on the basis of the above discussion, that the growth of a squall line in the meridional direction is a necessary result of the variation of the depth of the lower layer of the warm sector. Because of the rotation of the earth and the velocity of the fields of motion, the surface of inversion is, in general, not horizontal but it has a definite slope. Because this slope is rather sensitive to various factors that were considered to be negligible in the present treatment, no emphasis is placed on its determination from purely dynamical reasoning. For practical applications, this slope must be determined from aerological observations. However, a rough approximation will be given presently.

It will be assumed that the basic flow in the two atmospheric layers is strictly horizontal and does not change with height, and that geostrophic conditions are realized, so that the pressures in both layers may be expressed by the hydrostatic relation. Under these assumptions, and from figs. 1 and 2, it can be seen that

$$\begin{aligned} p &= g\rho' H + g\rho(h - z), \\ p' &= g\rho'(H + h - z), \end{aligned} \tag{24}$$

where p and p' are the pressures in the lower and upper layers, respectively, at a point whose height above the ground is z.

From the geostrophic relation and (24),

$$\begin{aligned} \frac{\partial P}{\partial y} &= g\rho' \frac{\partial H}{\partial y} + g\rho \frac{\partial h}{\partial y} = -\rho f u_0, \\ \frac{\partial P'}{\partial y} &= g\rho' \frac{\partial H}{\partial y} + g\rho' \frac{\partial h}{\partial y} = -\rho' f u_0', \end{aligned} \tag{25}$$

where u_0' is the constant velocity in the upper layer.

From (25), and from the definition of m,

$$m = -\frac{\partial h}{\partial y} = \frac{\rho u_0 - \rho' u_0'}{\rho - \rho'} \frac{f}{g}. \tag{26}$$

Because $\rho > \rho'$, the sign of the slope m depends upon the sign of the numerator. In order that the inversion layer may slope downward towards the center of the cyclone, as assumed in the previous treatment, the following inequality must be true:

$$\rho u_0 - \rho' u'_0 > 0.$$

It is understood that both ρu_0 and $\rho' u_0'$ are to be taken as the integrated values over the entire layers.

The main drawback to this method is that the vertical motions are not accounted for. These vertical motions introduce serious effects on this result, and the simplified formula (26) can hardly be justified.

The main points of defect in the treatment presented in the present paper are (1) the neglect of the transverse motion, (2) the assumption of a constant acceleration for the cold front, and (3) the assumption of a flat horizontal ground. These factors need further investigation. However, a word may be said about each of these factors.

Transverse motion is expected to arise because of the effect of the Coriolis force. This may introduce some modifications in the mathematical treatment of the problem. It is expected that, because of this motion, and because of the basic slope of the inversion layer, some of the disturbed fluid will overflow to the sides of the basic current. This overflow may, therefore, result in an extension of the computed length of the jump, and in a decrease in the "head" of the broken fluid. Thus, while the resulting squall line may spread more than predicted, its intensity may decrease.

The assumption of a constant acceleration for the cold front is hardly realized in nature. However, this assumption may, in the practical application, be overcome by solving the problem stepwise. As a matter of fact, it is expected that there may be different accelerations at different parts of the cold front. To correct for this, one may divide the warm sector into different strips in the Z-y direction and treat each strip separately.

The slope of the ground may introduce still some further modifications on the solution. With a sloping ground, the problem becomes analogous to that of the "breakers" which are formed when ocean waves approach a sloping beach. The general effect of this slope is to cause the waves to break earlier and to have a greater "head" (see Stoker, 1948).

It is quite possible that a similar effect may result in the atmosphere when the elevated fluid overtakes the warm front before it breaks. It may be surmized that some of the squall lines that are sometimes observed ahead of the warm front are caused by such a process. The mathematical treatment of this problem, however, is further complicated by the fact that the cold air below the surface of the warm front may be disturbed, and two "waves" may result, one at the surface of the inversion layer, and the other at the warm front surface.

Acknowledgments.—The writer wishes to express his thanks to Prof. James Miller of New York University for his valuable remarks. The writer is also indebted to Dr. M. Tepper of the U. S. Weather Bureau for his invaluable suggestions.

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