A NOTE ON THE PERIODICITY OF SOME FRONTAL PRECIPITATION

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Introduction.—It has not infrequently been observed that some types of precipitation appear in bands. To an observer on the ground, these bands appear to be more or less parallel, and manifest a certain degree of periodicity.

In recent years, radar photography has contributed substantially to these observations. The Massachusetts Institute of Technology weather-radar research group has taken photographs of some rain storms. Some of those photographs show these periodic parallel bands to a fair degree of clarity. As examples of these photographs, reference may be made to figs. 9 and 13 of Ligda (1951).

Ligda (1948) has also devised a most interesting method for observing precipitation bands by radar, and following their development in both time and space. His method consists of photographing the storm area at regular, short intervals of time. The separate photographs are then combined in one motion picture. Close and careful observation of the motion picture of one storm has been possible for the present writer, and quantitative measurements of the meteorological elements relating to that storm were available. These measurements make it possible to test any hypothesis that may be advanced in an attempt to explain the nature of these precipitation bands. The particular case for which quantitative data are available was a case of pre–warm-front precipitation. The bands seemed to be more or less parallel to the warm front. There was a marked frontal discontinuity in the temperature and wind fields.

Hypothesis.—A casual look at the radar pictures or the radar motion pictures of these rain storms does not fail to remind the observer of their wave-like behavior. These bands look very similar to an oceanic surface on a stormy day. It is true that the precipitation bands manifest a great degree of irregularity in the shape of the individual bands and in their recurrence; but this is also true in the case of an oceanic surface.

The writer wishes, therefore, to advance the hypothesis that these pre-frontal precipitation bands are a wave phenomenon, *i.e.*, that they are as much of a wave phenomenon as oceanic surface waves. According to this hypothesis, these bands are associated with gravitational wave oscillations at the frontal surface. However, they are far from being simple sinusoidal waves. Each band is a complex wave by itself. It is a "group" of waves. The group is composed of a great number of smaller "cells" of precipitation. The velocity of the group is different from the velocity of its constituent cells. The former travels with the group velocity, and the latter with the phase velocity. It will be shown that, in the case under consideration, the group velocity is smaller than the phase velocity. Hence if attention be fixed on a particular cell, it is seen to advance through the group, gradually dying out as it approaches the front, while its former place in the group is occupied in succession by other cells which have come forward from the rear.

A formula for the velocity of progress of the precipitation bands.—It has been indicated that the precipitation bands are assumed to be caused by an oscillatory disturbance that travels at the frontal surface. The mathematical problem is then reduced to the deduction of the group velocity of wave motion at a frontal surface.

To idealize a simple model, it is assumed that the frontal surface is plane and horizontal. The two air masses separated by it are assumed to be very deep, and homogeneous. The compressibility of the air, the rotation of the earth and its spherical shape are neglected. Frictional forces are also neglected.

Let the upper fluid have the constant horizontal velocity u' and density ρ', and let the corresponding quantities for the lower fluid be u and ρ. u is assumed to be parallel to u'.

Then it is easy to show [see, for instance, Lamb (1932)] that the phase velocity for waves, that are propagated at the frontal surface, is given by the following formula:

$$c = \alpha \pm [(g\beta/m) - \gamma]^{\frac{1}{2}}, \qquad (1)$$

where c is the phase velocity, g the acceleration of gravity, and m is given by $2\pi/\lambda$ when λ is the wavelength. Upon using the equation of state for air, and assuming the two layers to be isothermal, we find that the quantities α, β, and γ have the following values:

$$\alpha = (T'u + Tu')/(T + T'),$$

$$\beta = (T' - T)/(T' + T),$$

and

$$\gamma = [TT'/(T' + T)^2](u' - u)^2.$$

Formula (1) may serve to give an approximate estimate for the speed of the individual component cells which constitute the precipitation band. However, as stated above, these individual cells have no great significance by themselves. They appear at the rear of the band, and disappear at its front. They are, therefore, only important insofar as they combine together and form the group as a whole. It is seen from (1) that the phase velocity is a function of the wavelength. Hence, the atmosphere acts as a dispersive medium in regard to these cells. Each individual cell has a velocity of its own, corresponding to its wavelength.

The group velocity may be computed from the formula

$$c_g = d(mc)/dm. \qquad (2)$$

Upon substitution from (1) in (2), the following formula is obtained:

$$c_g = c - (g\beta/2m)(c - \alpha)^{-1}. \qquad (3)$$

In the derivation of this formula, the negative sign of (1) was neglected, since it is good for retrograde waves only.

It is suggested that (3) is proper for the computation of the velocity of progress of the precipitation bands under consideration, subject to the errors introduced by the idealized conditions.

It can easily be shown that the mechanical energy of wave motion is propagated at the rate given by (3) [see, for instance, Lamb (1932)]. The precipitation bands therefore travel with the mechanical energy of their medium.

Because α is positive as long as u and u' are positive, and if the wave is progressive so that only the positive sign of (1) is valid, it follows that the second term on the right of (3) is positive. Hence, it follows that

$$c_g < c. \qquad (4)$$

For these bands, therefore, the group travels with a velocity that is smaller than the velocity of its individual components. This result was anticipated in the discussion given above.

Numerical verification.—Numerical data are available for one storm, which affected Boston on 15–16 December 1947. This storm was photographed *via* the radar at the Massachusetts Institute of Technology, and detailed measurements were made for it. The cyclone developed in the Gulf of Mexico, and moved northeastward into New England. Two low-pressure centers formed, while it was still somewhat southwest of Boston. These were joined by a bent occluded front. The precipitation in the Boston area was associated with the warm front of the easternmost low-pressure center.

When the films of the storm were viewed at high speed, distinct parallel bands of precipitation appeared on the PPI scope image at maximum range southwest of Boston, and moved off to the northeast. The direction of movement was on a line normal to the major axis of the precipitation bands, and the bands were approximately equal distances apart.

The average speed and wavelengths of the precipitation bands were measured at hourly intervals from the film, and the 0300Z Nantucket rawin and raob data were used for the analysis.

For more detailed description of the storm, the reader is referred to Ligda (1948). The following numerical data are taken from that paper:

Average wavelength of 6 measurements = 72 km;
Average wind velocity in layer 1000 ft thick below frontal surface = 15.6 m/sec;
Average wind velocity in layer 1000 ft thick above frontal surface = 11.2 m/sec;
Virtual temperature below frontal surface = 261K;
Virtual temperature above frontal surface = 267K;
Average velocity of progress of 3 precipitation bands = 21 m/sec.

In terms of the notation used in the present paper' the observed data are: $\lambda = 72 \times 10^5$ cm, $T' = 267$K, $T = 261$K, $u' = 11.2 \times 10^2$ cm/sec, $u = 15.6 \times 10^2$ cm/sec. And take g to be 980 cm/sec^2. Upon substitution of these values in (1), and then in (3), the following results are obtained: $c = 70$ m/sec, and $c_g = 26$ m/sec.

The computed period of recurrence for these bands is 46 min. The observational data give a period of 57 min.

The agreement between theory and observation is considered fairly close. It is therefore inferred that the group velocity offers a reasonably good approximation to the velocity with which the precipitation bands travel.

Discussion.—It may be noticed, on the basis of the above discussion, that the wave hypothesis may serve to explain the nature of the periodic pre-warm-front precipitation bands. It serves also to offer a formula by virtue of which their future displacement may be prognosticated with a fair degree of accuracy. The one point that needs some emphasis in the hypothesis is that the precipitation bands are far from being simple, individual sinusoidal trains of waves. They are of the nature of complex wave packets, which may be considered to be composed of sinusoidal waves. This last consideration may be looked at as a purely mathematical tool, to enable the analyzer to scrutinize the phenomenon. To it may also be attributed some physical significance, by the consideration that the disturbing factor is capable of producing an infinite number of pure sinusoidal waves. The medium is capable of transmitting the totality of these infinite waves. But these waves interfere with each other, and produce the complex packet. Since the medium is dispersive, the packet travels at the group velocity, whereas each of its constituent components travels with the phase velocity relevant to its wavelength.

Because the verification was limited to one observed case only, the writer does not wish to overemphasize the numerical agreement between theory and observation. Further tests of (3) must await some further quantitative observations.

It is hoped that this note may serve to enhance further numerical and theoretical studies of similar cases.

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