The Spiral Bands of a Hurricane: A Possible Dynamic Explanation

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ABSTRACT

In this paper an attempt is made to explain the shape and behavior of the spiral bands of a hurricane. The main hypothesis is that the bands are associated with gravitational waves of finite amplitude propagating at the interface of a high-level inversion. An external source of disturbance is postulated in the form of a fresh surge of air at the exterior region of the hurricane. It is shown that this mechanism leads to the formation of bands of the required shape. The spiral bands are therefore related to the squall lines of temperate latitudes. Two numerical examples are worked out to illustrate the proposed mechanism.

1. Introduction

During the last two decades observations of cloud and rain distributions in hurricanes have become available, either through the use of radar, by airplane reconnaissance, or by aerial photographs taken from satellites. One of the most striking features that may be discerned from such observations is the appearance of large-scale bands that have a spiral shape. They move in the same direction of the wind circulation spiraling cyclonically inward toward the center (Dunn, 1951). Thunderstorms, severe gusty winds, and even tornadoes may occur during their passage.

Attention was first called to these bands by Wexler (1945, 1947) who made a detailed study of the structure of three hurricanes. He suggested that the mature hurricane has a band-like structure. He also suggested that the spiral bands may be caused by some kind of motion which follows the streamlines. Riehl (1951) suggested that a possible explanation for these bands is the development of internal waves. Following this suggestion, the present writer (1953) attempted to study the small free vibrations of a hurricane. It was found that the convective pattern caused by these vibrations consists of individual separated cells which, in general, may not lie on a continuous wave front with a spiral shape.

Recently many attempts have been made to study the spiral bands in more detail, and to propose some working mechanism that may explain their main features. For an excellent review of these attempts reference may be made to Donaldson and Atlas (1963). Ligda (1955b) attributed the formation of these bands to selective growth of widespread continuous rain originating at high levels. Kessler and Atlas (1956) assumed some banded structure of horizontal convergence. Tepper (1958) proposed a model in which a disturbance in the flow around an inner ring of convergence surrounding the eye is propagated outward as a gravity wave. His model, however, would give the bands a closed, rather than an open, spiral shape. Senn and Hiser (1958, 1959) assumed some kind of a point disturbance moving around the eye and giving rise to gravity waves. Atlas *et al.* (1963) assumed that a band is a plume of growing stratiform precipitation, released from a convective source cloud at the head of the band. However, they felt the need to explain the process which initiates the convective source clouds, their orderly arrangement in time and space and their propagation at velocities different from the wind field. Donaldson and Atlas (1963) concluded by stating that "in each case the essential but unexplained feature is a disturbance capable of generating new clouds, propagating outward in an approximately radial direction with respect to the hurricane eye."

It therefore seems to the present writer that a theoretical mechanism proposed to explain the bands must be capable of accounting for the following main features:

1) The shape of a band is an open spiral.

2) The bands move around the eye with speed that is different from, and usually greater than, the speed of the basic flow. Ligda (1955a) has measured the speeds of some cells and found them to be about twice that of the wind.

3) The bands seem to grow radially at a great rate. Tatehira (1961) found that the echo of the cells moved radially outward at a phenomenal rate of about 100 km hr^{-1}.

4) The individual bands have a short life span, rarely exceeding one or two hours (Senn and Hiser, 1958, 1959).

5) The bands are mostly observed in mature hurricanes when they approach land (Wexler, 1947; Watanabe, 1963).

6) The bands are attended by severe weather that is reminiscent of that associated with the temperate latitude squall lines (Ligda, 1955b).

The present article is an attempt in this direction.

2. The theoretical model

The theoretical model dealt with in the present article incorporates some simplifying assumptions that are made in order to facilitate the mathematical analysis, the main object being to propose a simple mechanism that may yield an explanation of the main features of the spiral bands, and which may be feasible at least as a first approximation.

It is assumed that a hurricane consists of two distinct regions: the eye which is circular in shape, and the outer region which is symmetrical around the eye. The model is that of a stationary Rankine combined vortex. The air inside the eye rotates around its geometrical axis like a solid body. The wind velocity in the outer region falls off with distance according to the hyperbolic law. This is the same wind distribution given by Deppermann (1947) for an idealized typhoon. Moreover, the wind velocity is assumed to be independent of height.

The vertical structure of the hurricane is assumed to consist of two distinct layers. The lower layer takes part in the circulation according to the mode just described. The upper layer, which may consist of all the atmosphere above the lower layer, is assumed to remain appreciably unaffected by the disturbances that may take place in the lower layer.

Initially it is assumed that the circulation is in steady state. At a certain stage in the lifetime of the storm a disturbance is introduced at a certain region. It may be conjectured that this disturbance may be caused by some new air mass that is injected into the circulation at the outer region. This, however, is not a particularly trenchant assumption. It is a well known fact of observation that hurricanes do receive some surges of new air masses during their lifetime. Various attempts have already been made to study the nature, frequency and amount of inflows, and the sector through which they penetrate. Dunn (1951), for example, has stated that during or immediately following recurvature N_p transitional polar air may be encountered. Even fronts may be observed at the stage of decay. Watanabe (1963) has analyzed a number of typhoons and found that cold air is usually advected into the circulation. He also noticed the existence of a relation between fresh surges and the formation of the spiral bands.

A fresh surge of air is not expected to be simply drifted by the pre-existing circulation, but it usually has a somewhat different motion. The shape of the leading edge of the surge and its motion may vary from one case to the other. However, for the sake of simplicity, it will be assumed in the present model that a certain radial sector of the air in the outer region acquires some prescribed motion relative to the pre-existing circulation, the motion being caused by the re-entrant surge. Because of this motion a disturbance is initiated which propagates through the previously undisturbed air mass making up the hurricane. This disturbance may reveal itself in the form of a gravitational wave at the upper surface of the lower layer. If the initial conditions are favorable, a gravitational wave of elevation may be produced, which propagates at the interface and finally breaks. The breaking is usually associated with turbulence and the release of latent instability. This will manifest itself in the form of a squall line circulating inside the hurricane. If the initially disturbed air mass performs an oscillatory motion, rather than continues to advance in the same direction, the process may be expected to repeat itself, and a new squall line may be associated with every forward motion. Such squall lines will be separated by waves of depression which advance behind the squall lines and finally help in their decay.

It will be shown in the present article that the squall lines formed in this way have a spiral shape. They may start at the periphery of the eye and grow outward with great rapidity. While growing in this manner they circulate around the hurricane with a velocity greater than the velocity of the basic flow.

The mechanism proposed in the present article is similar to that proposed by the present writer (1949) for the formation of squall lines behind the cold front, and by Tepper (1950) for the formation of pre-cold front squall lines. The growth of squall lines in a straight flow was first described by Brunk (1949), and studied analytically by Abdullah (1954a). In the present case the curved path of the basic flow is introduced.

3. Mathematical analysis

Fig. 1 is a schematic representation of the idealized model showing both horizontal and vertical cross sections.

Take a cylindrical polar system of coordinates such that the origin is at the geometrical center of the eye, and the angle θ is measured, in the positive direction, from an arbitrarily chosen fixed line OX. The axis of z is vertical and pointing upwards.

Let the air masses affected by the circulation be assumed to consist of two incompressible layers. The lower layer has the horizontal ground for a lower boundary. Its upper surface is at a height $h(r)$ which is a function of r only. When the fluid is not disturbed $h(r)$ is a monotonous increasing function of r as indicated in Fig. 1b. The density of the fluid in this layer is ρ. Let the upper layer be resting on the lower layer, and let its upper surface be at a height H above the ground. This height may be taken to be equal to that of the homogeneous atmosphere relevant to the case under con-

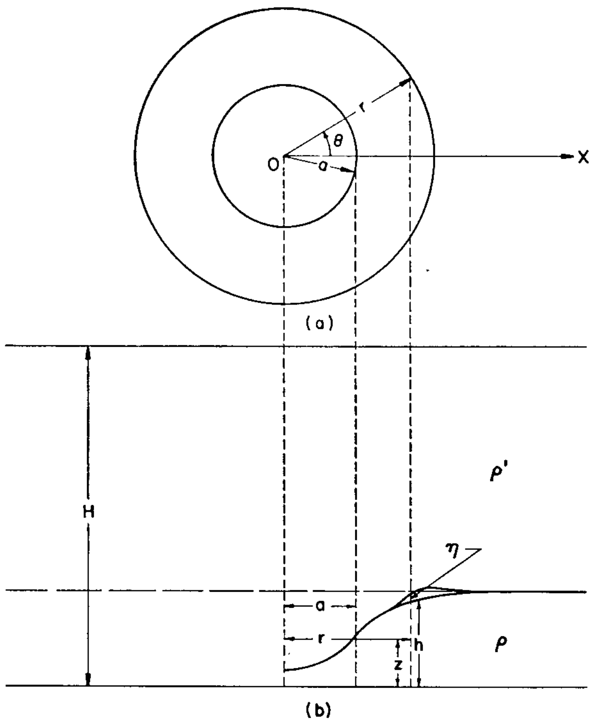

FIG. 1. Schematic representations of: (a) horizontal cross section of the idealized model. O is the center of the eye, a its radius. OX is an arbitrarily fixed direction, r is the radius vector to a point in the exterior region and θ is the angular distance measured from OX in the positive direction; (b) vertical cross section through the eye, showing the two layers. h is the undisturbed height of the lower layer, η is a displacement caused by the disturbance, H is the height above ground level of the free surface of the upper layer, ρ and ρ' are the densities of the two layers and z is the vertical coordinate of a point inside the lower fluid.

sideration. Let the density of the fluid in this layer be ρ'. The upper layer may be assumed to remain not appreciably affected by disturbances initiated in the lower layer, and the height H may be assumed to be independent of r and θ. Friction, the rotation of the earth and its spherical shape are neglected.

It is further assumed that the radial component of velocity is small in comparison with the tangential, so that it may be neglected. It is admitted that this assumption is only made to simplify the mathematical analysis. It may be justified by the conjecture that the pressure gradient adjusts itself in such a way as to admit no radial velocities.

Furthermore, it will be assumed that the hydrostatic relation holds since it has been shown by Haurwitz (1935) that this relation remains valid for storms of the nature under consideration.

The tangential component of the wind velocity in the undisturbed lower layer is assumed to be given by the relations

$$\left.\begin{aligned} u_0 &= \Omega r \quad o \leq r \leq a \\ u_0 &= \frac{\aleph}{r} \quad a \leq r \leq \infty \end{aligned}\right\}, \tag{1a}$$

where Ω and $\aleph$ are constants and r is the radius vector. If there is no slipping at the periphery of the eye, Ω and $\aleph$ are connected by the relation

$$\aleph = \Omega a^2. \tag{1b}$$

It will now be assumed that the lower layer is disturbed such that its upper surface is displaced by η from the original height h. η may be a function of θ, r and t.

Under the present assumption, the equations of motion for the lower layer take form [see, for example, Lamb (1945)],

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{r\partial\theta} = -\frac{1}{\rho}\frac{\partial p}{r\partial\theta}, \tag{2a}$$

$$-\frac{u^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial r}, \tag{2b}$$

$$0 = -\frac{\partial p}{\partial z} - g\rho, \tag{2c}$$

while the equation of continuity is

$$\frac{\partial u}{r\partial\theta} + \frac{\partial w}{\partial z} = 0, \tag{3}$$

where u and w are the tangential and vertical components of velocity respectively, p is the pressure and g is the acceleration of gravity.

The boundary conditions are

$$w = 0 \text{ at } z = 0, \tag{4a}$$

and

$$\frac{d}{dt}(h+\eta-z) = 0 \quad \text{at} \quad z = h+\eta, \tag{4b}$$

where

$$\frac{d}{dt} = \frac{\partial}{\partial t} + u\frac{\partial}{r\partial\theta} + w\frac{\partial}{\partial z}.$$

Condition (4b) may be written explicitly as

$$\frac{\partial\eta}{\partial t} + u\frac{\partial\eta}{r\partial\theta} = w \quad \text{at} \quad z = h+\eta. \tag{4c}$$

Upon integrating (3) with respect to z between the ground and $h+\eta$, and making use of the first boundary conditon (4a), we obtain

$$-w\Big|_{h+\eta} = \frac{\partial}{r\partial\theta}\int_0^{h+\eta} u dz - u\frac{\partial}{r\partial\theta}(h+\eta).$$

Assuming that u is independent of z, performing the integration in the last equation, and making use of the secondary boundary condition as given in (4c), the following equation results:

$$\frac{\partial\eta}{\partial t}+\frac{\partial}{r\partial\theta}(u\eta)=-\frac{h\partial u}{r\partial\theta}. \qquad (5)$$

From the hydrostatic relation given in (2c), it follows that the pressure at a point inside the lower layer may be given by

$$p=g\rho(h+\eta-z)+g\rho'(H-h-\eta). \qquad (6)$$

Hence,

$$\frac{\partial p}{\partial\theta}=g(\rho-\rho')\frac{\partial\eta}{\partial\theta}, \qquad (7)$$

h being independent of θ by assumption.

Upon inserting (7) in (2a) the latter takes the form

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{r\partial\theta}+g'\frac{\partial\eta}{r\partial\theta}=0, \qquad (8)$$

where

$$g'=g\frac{\rho-\rho'}{\rho}. \qquad (9)$$

The second equation of motion (2b) becomes

$$\frac{u^2}{r}=g'\frac{\partial}{\partial r}(h+\eta). \qquad (10)$$

For the undisturbed steady case, $\eta=0$ and $u=u_0$, and (10) thus reduces to

$$\frac{u_0^2}{r}=g'\frac{dh}{dr}. \qquad (11)$$

The height of the upper surface of the lower layer may be obtained from Eq. (11) by integration with respect to r after making use of relations (1). Thus for the eye

$$h-h_a=\frac{\Omega^2}{2g'}(r^2-a^2),\quad 0\leq r\leq a, \qquad (12)$$

and for the outer region

$$h-h_a=\frac{\Omega^2a^4}{2g'}\left(\frac{1}{a^2}-\frac{1}{r^2}\right),\quad a\leq r\leq\infty, \qquad (13)$$

where h_a is the height of the surface at the periphery of the eye.

Upon defining the variable c by the relation

$$c^2=g'(h+\eta), \qquad (14)$$

and inserting it in Eqs. (5) and (8), these become

$$2\frac{\partial c}{\partial t}+c\frac{\partial u}{r\partial\theta}+2u\frac{\partial c}{r\partial\theta}=0, \qquad (15)$$

and

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{r\partial\theta}+2c\frac{\partial c}{r\partial\theta}=0. \qquad (16)$$

Addition and subtraction of these two equations result in the following:

$$\left[\frac{\partial}{\partial t}+(u+c)\frac{\partial}{r\partial\theta}\right](u+2c)=0, \qquad (17)$$

$$\left[\frac{\partial}{\partial t}+(u-c)\frac{\partial}{r\partial\theta}\right](u-2c)=0. \qquad (18)$$

Eqs. (17) and (18) have the same form as those for long waves in a straight flow. A detailed discussion of these equations has been given by a number of authors. See, for example, Abdullah (1949) or Freeman (1951) and especially Stoker (1948, 1957). There is one difference, however, between the case under consideration and the cases treated by the authors referred to, for in the present case the number of independent variables is three, namely t, θ and r. Nevertheless, this does not introduce any further complications in the mathematical analysis. Because the assumption has been made that radial velocities may be neglected, the variable r is reduced to a parameter. The fluid may be conceived as consisting of a greater number of coaxial cylindrical rings. The motion in each one of these rings, being unaffected by the neighboring rings, may be treated separately. The general motion may then be constructed by combining together the totality of of the motions in these rings. This method is mathematically feasible and consistent with the present assumptions. Basically the same method has been followed by the present author (1954a) in discussing straight flows that may depend upon the transverse direction. The same method has also been suggested by Stoker (1953, 1957) and used by Whitham (1953).

Eqs. (17) and (18) give the following two sets of characteristics, with the corresponding Rieman invariants:

$$C_1:\quad \frac{rd\theta}{dt}=u+c, \qquad (19)$$

$$u+2c=k_1,\ \text{a constant}, \qquad (20)$$

and

$$C_2:\quad \frac{rd\theta}{dt}=u-c, \qquad (21)$$

$$u-2c=k_2,\ \text{a constant}. \qquad (22)$$

Upon writing $d\psi$ for $rd\theta$, Eqs. (19) and (21) may be written as

$$\frac{d\psi}{dt}=u+c, \quad \text{for} \quad C_1, \tag{23}$$

$$\frac{d\psi}{dt}=u-c, \quad \text{for} \quad C_2. \tag{24}$$

The quantity ψ gives the horizontal displacement along the circumference of the ring under consideration.

The present technique is equivalent to inserting a vertical plate along a radius, cutting the hurricane and opening it up. Each ring is thus transformed into a straight channel whose length is $2\pi r$. The disturbance created at one end propagates like a simple wave until it interacts with the disturbances created by the other end. It will be assumed that the length of the channels are long enough to permit the following treatment.

It will be assumed that the leading edge of the surge, which was along the radius $\theta=0$, at time $t=0$ is given a certain prescribed motion. At the ring whose radius is r_i let the horizontal displacement along the circumference be $\xi(\theta,\tau;r_i)$, where τ is the time elapsing after the instant $t=0$. The parameter r is written explicitly to emphasize the fact that the motion may vary from one ring to the other. On the $\psi-t$ plane, Fig. 2, the path of the leading edge of the surge is represented by the curve $\xi(\tau)$. The first infinitesimal disturbance would be propagated along the straight characteristic given by

$$C_{10}{}^i: \left.\frac{d\psi}{dt}\right)_0 = u_0{}^i + c_0{}^i.$$

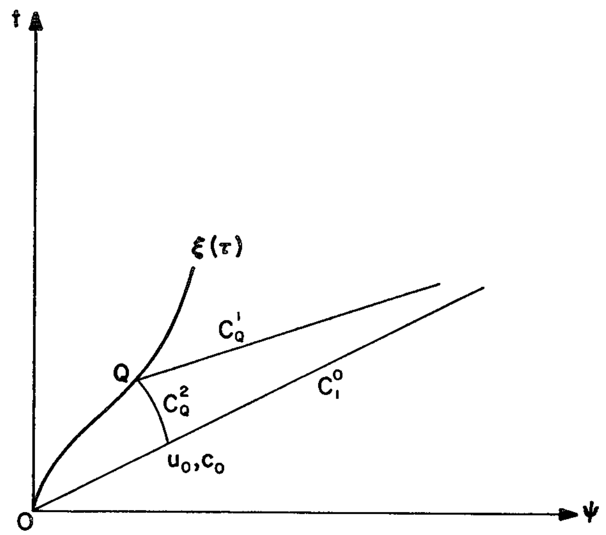

FIG. 2. A plot of the characteristics on a $\psi-t$ plane. $\xi(\tau)$ is the path of the leading edge of the surge. Q is any point on this path. The two characteristic lines issuing from Q are $C_Q{}^1$ and $C_Q{}^2$. $C_1{}^0$ is the path of an initial infinitesimal disturbance.

A straight characteristic issuing from the point Q on the curve has the slope

$$C_{1Q}{}^i: \left.\frac{d\psi}{dt}\right)_Q = u^i(\tau)+c^i(\tau),$$

while along the curved characteristic passing through Q we have

$$u^i(\tau)-2c^i(\tau)=u_0{}^i-2c_0{}^i.$$

Hence the slope of $C_{1Q}{}^i$ is given by

$$\left.\frac{d\psi}{dt}\right)_Q = \tfrac{3}{2}u^i(\tau)-\tfrac{1}{2}u_0{}^i+c_0{}^i. \tag{25}$$

The slope of this straight line may also be found from

$$\left.\frac{d\psi}{dt}\right)_Q = \frac{\psi-\xi(\tau)}{t-\tau}.$$

Hence,

$$\psi-\xi(\tau)=(t-\tau)[\tfrac{3}{2}u^i(\tau)-\tfrac{1}{2}u_0{}^i+c_0{}^i]. \tag{26}$$

If the motion of the surge is forward relative to the basic flow the C_1 characteristics converge, intersect and form an envelope. To find the parametric equations of this envelope, differentiate (26) partially with respect to τ, set the result equal to zero and solve the resulting equation simultaneously with Eq. (26). The following equations are found to represent the envelope:

$$t_e=\tau+\frac{1}{\frac{3}{2}u^i(\tau')}[\tfrac{3}{2}u^i(\tau)-\tfrac{1}{2}u_0{}^i+c_0{}^i-\xi'(\tau)], \tag{27}$$

and

$$\psi_e=\frac{1}{\frac{3}{2}u^{i\prime}(\tau)}[\tfrac{3}{2}u^i(\tau)-\tfrac{1}{2}u_0{}^i+c_0{}^i-\xi'(\tau)] \times[\tfrac{3}{2}u^i(\tau)-\tfrac{1}{2}u_0{}^i+c_0{}^i]+\xi(\tau), \tag{28}$$

where the primes denote differentiation with respect to τ.

The cusp of the envelope represents the first point where an elevation wave, described by the previous equations, will have an infinite slope and where the breaking starts. This point lies on the line $\tau=0$ (see Stoker, 1948). Hence breaking will start at the point (t_b,ψ_b) as given by

$$t_b{}^i=\frac{1}{\frac{3}{2}u^{i\prime}(0)}[\tfrac{3}{2}u^i(0)-u_0{}^i+c_0{}^i-\xi'(0)], \tag{29a}$$

$$\psi_b{}^i=\frac{1}{\frac{3}{2}u^{i\prime}(0)}[\tfrac{3}{2}u^i(0)-\tfrac{1}{2}u_0{}^i+c_0{}^i-\xi'(0)] \times[\tfrac{3}{2}u^i(0)-\tfrac{1}{2}u_0{}^i+c_0{}^i], \tag{29b}$$

where the index i has been written to show that these quantities may vary according to the ring considered.

Eqs. (29) give the time and location on the $\psi-t$ plane of the point where breaking starts. It is presumed that the corresponding point on the physical plane is also the point where the squall line is first observed. Because both $t_b{}^i$ and $\psi_b{}^i$ are functions of r, the squall line will appear in the shape determined by the nature of the functional relationship assumed in the various variables on the right hand sides of these equations.

4. Numerical example

It will be assumed that the leading edge of the surge executes a motion described by

$$\xi^i(\tau)=A^i(1-\cos\omega^i\tau)+u_0{}^i\tau, \tag{30}$$

where A^i and ω^i may be functions of r. This function makes the fluid right in touch with the surge, and which was at the time $t=0$ on the radius $\theta=0$, execute an oscillatory motion relative to the basic flow. This motion is characterized by forward acceleration during the first quarter of a period. The function given in (30) is the same as that used by the present writer (1955) to discuss the formation of the atmospheric solitary waves. It has also been used by Greenfield and Miller (1963) to discuss atmospheric breakers.

Let us consider the resulting disturbance due to the first quarter of a period, i.e., during the time $0\leq\tau\leq\pi/2\omega^i$. It follows from Eq. (20) that at the instant $\tau=0$, one obtains

$$\left.\begin{aligned}\xi'(0)&=u^i(0)\\ u^i(0)&=u_0{}^i\\ u^{i\prime}(0)&=A^i\omega^{i2}\end{aligned}\right\}. \tag{31}$$

Upon inserting these values in Eqs. (29), there results

$$t_b{}^i=\frac{c_0{}^i}{\frac{3}{2}A^i\omega^{i2}}, \tag{32a}$$

and

$$\psi_b{}^i=\frac{c_0{}^i(u_0{}^i+c_0{}^i)}{\frac{3}{2}A^i\omega^{i2}}. \tag{32b}$$

Eqs. (32) give the time and displacement of the point associated with the birth of the squall line which is here identified with the bands. Because the quantities on the right-hand sides of these equations are functions of r it may be inferred that the band does not start at all radii at the same time. It starts rather either at the innermost ring bordering at the periphery of the eye or at some outer ring taking part in the vortex circulation, the preferable point for first breaking being determined by the nature of the functional dependence upon r. When it starts at the innermost ring it grows outward while rotating inside the storm. When it starts at an outer ring it grows inward.

In the following, two numerical cases have been worked out. In the first case we assume, $a=20$ km, $\Omega=10^{-3}$ sec^{-1}, and hence $\aleph=4\cdot10^9$ cm^2 sec^{-1}. The mean temperature of the lower layer is 21C, and that of the upper 27C. Hence $g=20$ cgs units. ω is assumed to be a constant having the value $2\cdot10^{-3}$ sec^{-1}. This corresponds to a period of oscillation equal to $\pi\cdot10^3$ sec. A is assumed to have the value of 1 km, a constant which does not depend upon r. The upper fluid is assumed to be just touching the ground at the center of the eye. From Eq. (12) it follows that the height of the interface at the periphery of the eye is $h_a=1$ km.

Fig. 3 shows the computed values of the basic undisturbed velocity u_0, the height h_0 and the critical velocity c_0 as computed from Eqs. (1), (13), (14) and (15), respectively.

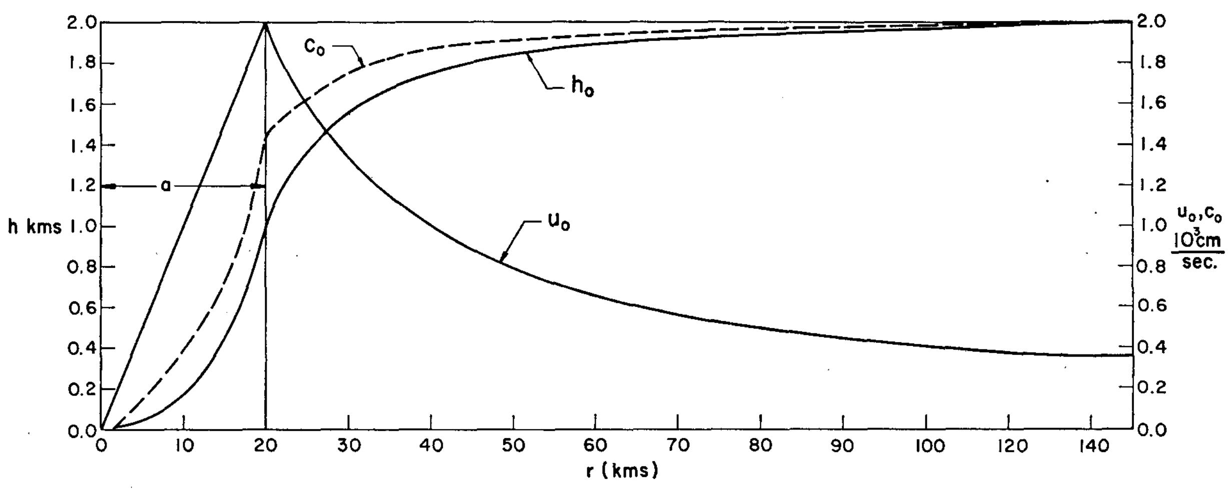

FIG. 3. A plot of the three undisturbed variables h_0, u_0 and c_0 against the radial distance from the eye of the hurricane where h_0 is in km, and u_0 and c_0 in cm sec^{-1}.

Fig. 4 shows the formation of the squall line. Curve I shows the location of the starting points of breaking, the time of breaking at each ring being indicated on the curve. Eqs. (32) were used to compute these values. Curve II shows the squall line after the breaking has been completed along a radius of 100 km, the various segments of the squall line being assumed to move with the constant speed $u_0^i+c_0^i$.

In the present case it may be seen that the squall line appears first at the periphery of the eye, and it grows outwards as it rotates cyclonically in the same sense as the basic flow. It may be seen from the indicated values of t_b that once the squall line appears, it grows outward at a great rate. Thus in the present case it takes less than 20 min to advance from the periphery of the eye to a point at the radius 100 km.

Curve III of Fig. 4 shows the shape and location of the same band at a time $\pi \cdot 10^3$ sec (nearly one hour) after the instant of complete birth, i.e., after curve II.

In the second numerical case the same values of the first case have been taken, with the exception of A. A has been assumed to be a linear function of r subject to the relation $A=0.05\ r$. Fig. 5 gives the computed bands corresponding to this case. It may be seen that in this second case the band appears first at the outer ring and grows inward very rapidly.

In both cases it may be seen that the final shape of the band are spirals resembling the shape observed

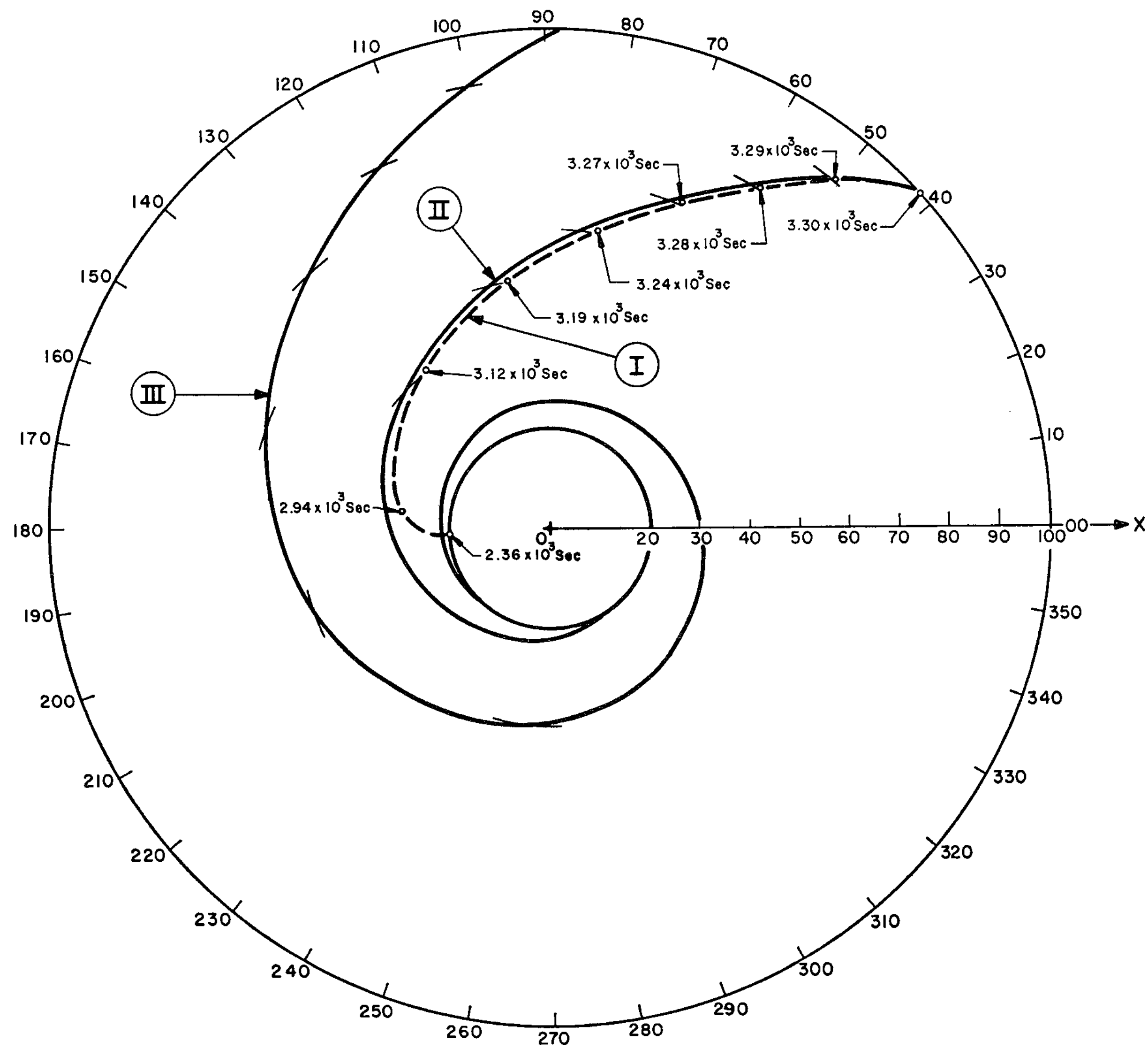

FIG. 4. Formation of the spiral band as computed from example 1 for the values $A=1$ km and $w=2\cdot10^{-3}$ sec^{-1}. OX is the initial location of the leading edge of the surge. Curve I shows the location of the starting points of breaking. The time of breaking at each ring, as measured from the instant OX starts to move, is indicated on the curve. Curve II shows the band after the breaking has reached the ring of radius 100 km. Curve III shows the band at a time of one period after curve II.

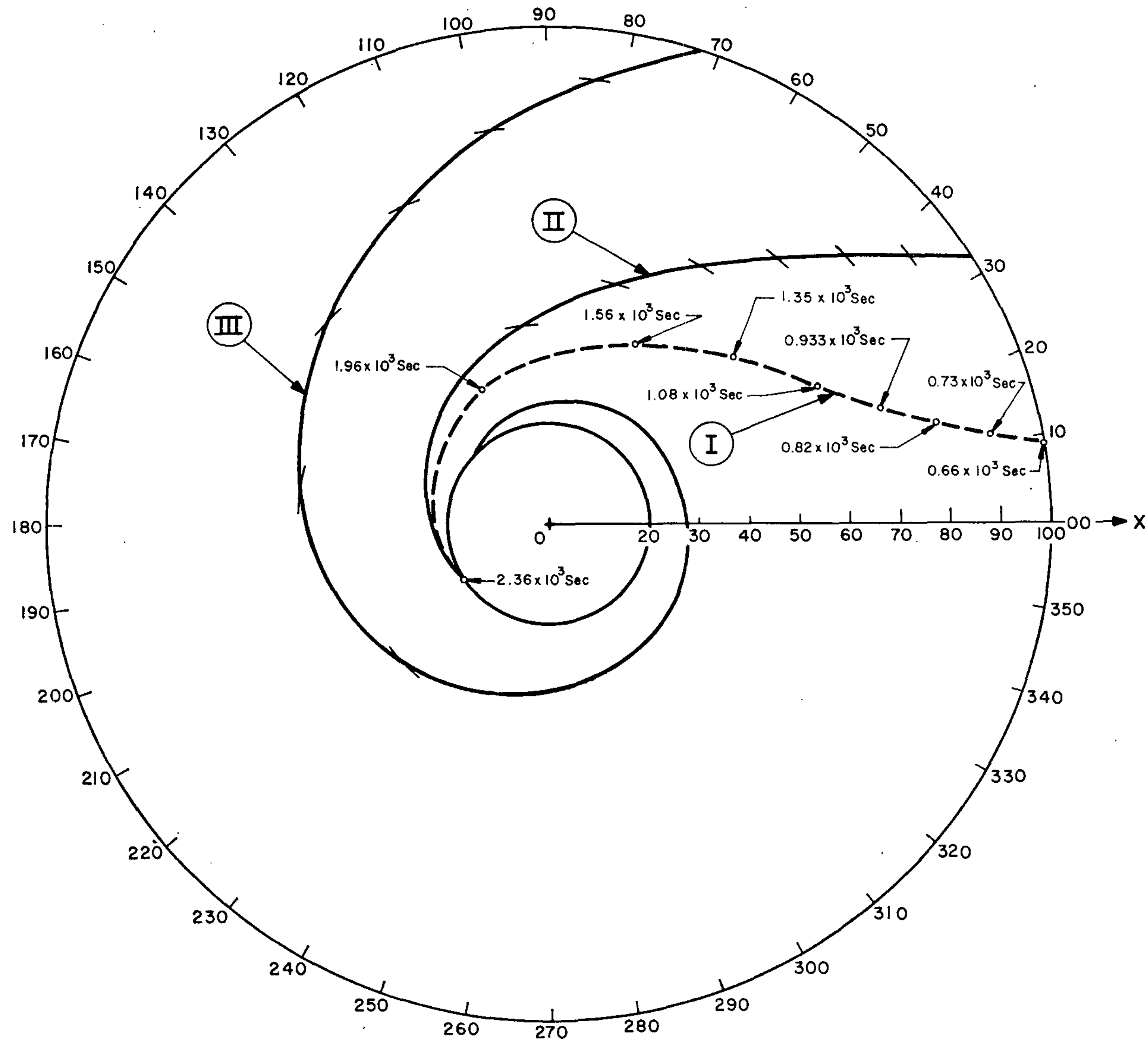

FIG. 5. Same as Fig. 4 with $A=0.05r$ and $w=2\cdot 10^{-3}$ sec^{-1}.

in nature, and that they grow radially with great rapidity.

5. Discussion and further remarks

On the basis of the analysis cited in the present paper it may be concluded that the spiral bands of a hurricane are closely related to the squall lines of temperate latitudes. The same mechanism is capable of accounting for all the main features listed in the introduction to the present article, as may immediately be inferred. The fact that the bands have, usually, a short life span follows from the fact that the function described in Eq. (30) would call for a depression wave following the squall line. This wave is known to catch up with the squall line and acts to destroy it (Stoker, 1948).

Before concluding, the following remarks may be made:

a) In the proposed mechanism a surge of cold air was assumed to penetrate into the outer region and give rise to the subsequent motion. This, however, is not a necessary requirement. It suffices, for our purpose, to assume that the air in a certain sector acquires an oscillatory motion according to the law prescribed in Eq. (30). The fresh cold air would then simply act to cause that sector to oscillate. If this picture is adopted, it may be possible to compute the function ω relevant to the initial conditions. This function cannot be very different from the angular frequency with which the hurricane vibrates freely. These vibrations have been discussed by the present writer (1953).

b) In the numerical examples, the disturbed sector was assumed to perform one forward accelerated movement. This was found to be sufficient for the creation of one spiral band. If the disturbed sector is allowed to perform a few oscillations, it may be expected to result in a family of bands following each other, with regions of depression waves separating them. Observations seem to bear out this conclusion.

c) Because the flow inside a hurricane is bounded in space, an oscillating mechanism of the nature postulated here may give rise to waves in both directions. This are similar to a closed channel with disturbances being initiated at both ends. The interactions of these disturbances are complicated by the nonlinearity of the basic equations. Some cases of interest form the subject matter for a forthcoming communication.

d) The explanation of the nature of the spiral bands is of special interest in studying the energy budget of a hurricane. If it be established that they conform to what has been cited in the present communication, the results published by the writer in a previous article (1954b) would hold. This would seem to indicate that the bands feed on the mechanical energy of the hurricane, and thus act to destroy it.

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